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Investigating the effect of the deflection of the vertical on lidar observations[1]

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Abstract. Considerable materials have been generated that focus on the error analysis of light detection and ranging (lidar) derived coordinates through the direct georeferencing equation. One component of the equation, namely the deflection of the vertical (DOV), has been largely ignored within the literature. This rotational component serves to reconcile the ellipsoidal and local-level reference systems and is often considered to be insignificant. The sensitivity of lidar-derived coordinates to the deflection of the vertical was investigated through simulation. This is accomplished by deriving three-dimensional coordinates through the direct georeferencing equation and both ignoring and including the deflection of the vertical. Failure to consider this component was found to overcome commercially published horizontal accuracies at magnitudes of 34″, 35″, and 37″ for flying heights of 1000, 2000, and 3000 m, respectively, and vertical accuracies at 53″, 40″, and 35″ for flying heights of 1000, 2000, and 3000 m, respectively. Values of this magnitude are prevalent in mountainous environments and should not be ignored. Lastly, the unavoidable error existing in determinations of the deflection of the vertical was reported and was also determined to be significant with respect to the overall lidar error budget.

Résumé. Beaucoup de documentation a été générée concernant l’analyse d’erreurs des coordonnées dérivées du lidar par le biais de l’équation de géoréférencement direct. Dans la littérature, une des composantes de l’équation, la déviation de la verticale (DV), a été largement ignorée. La composante rotationnelle sert à réconcilier les systèmes de référence ellipsoïdal et local et est souvent considérée comme négligeable. La sensibilité des coordonnées dérivées du lidar à la déviation de la verticale a été examinée par le biais d’une simulation. Ceci est réalisé en dérivant des coordonnées tridimensionnelles au moyen de l’équation de géoréférencement direct et en ignorant et en incluant à la fois la déviation de la verticale. Le fait de ne pas considérer cette composante a permis de compenser les précisions horizontales publiées commercialement à des magnitudes de 34″, 35″ et 37″ pour des hauteurs de vol de 1000 m, 2000 m et 3000 m respectivement et les précisions verticales à 53″, 40″ et 35″ pour les hauteurs de vol de 1000 m, 2000 m et 3000 m respectivement. Des valeurs de cette magnitude sont fréquentes dans les environnements montagneux et celles-ci ne devraient pas être ignorées. Enfin, l’erreur inévitable existant dans les déterminations de la déviation de la verticale a été rapportée et déterminée comme étant également significative dans le budget global d’erreurs lidar.
[Traduit par la Rédaction]

Introduction

Light detection and ranging (lidar) technology has gained widespread acceptance because of its ability to rapidly acquire three-dimensional point data across terrain surfaces at high densities and with a high degree of accuracy. A lidar system is generally mounted on an airborne platform such as a fixed-wing aircraft or helicopter. As the platform experiences forward movement, a laser ranging device transmits light pulses that are reflected from the terrain surface and returned to the platform. The receiving optics of the system interrogate the returned signal and determine a two-way travel time. With knowledge of the speed of light, the two-way travel time is converted to a range. Contemporary lasers are capable of high-frequency observation rates that can exceed 200 kHz. To sample large areas of terrain within a single flight line, a scanning system rotates a mirror that directs the laser pulse across a swath beneath the aircraft. The positioning of the airborne platform is controlled by an inertial measurement unit used to provide aircraft orientation and a global navigation satellite system (GNSS) receiver used to provide the absolute position. The combination of the observations from each of these complementary technologies allows a three-dimensional point position to be determined on the physical surface of the Earth (Vaughn et al., 1996; Wehr and Lohr, 1999).

The combination of each of these observations requires the reconciliation of several different reference frames. This is performed through the direct georeferencing equation,

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originally developed in Vaughn et al. (1996) as follows:

$$[r_{\mathrm{Ground}}] = [r_{\mathrm{GNSS}}] + \mathbf{R}_{\mathrm{G}}\mathbf{R}_{\mathrm{INS}} \times \left([\boldsymbol{r}_{\mathrm{Ecc}}] + \mathbf{R}_{\mathrm{Boresight}}\mathbf{R}_{\mathrm{ScanAngle}}\left[r_{\mathrm{Range}}\right]\right) \qquad (1)$$

where r_{Ground} is the observed ground coordinate; r_{GNSS} is the observed absolute position on the aircraft platform at the GNSS receiver; $\mathbf{R}_{\mathrm{G}}$ is a rotation from the local-level frame to the ellipsoidal frame by the deflection of the vertical (DOV); $\mathbf{R}_{\mathrm{INS}}$ is a rotation from the body frame to the local-level frame; $\boldsymbol{r}_{\mathrm{Ecc}}$ is a vector of distances between the phase centre of the global positioning system (GPS) antenna and the transmission point of the laser measured in the aircraft body frame, commonly referred to as the eccentricity; $\mathbf{R}_{\mathrm{Boresight}}$ is a rotation from the laser scanning frame to the aircraft body frame by the boresight angles; $\mathbf{R}_{\mathrm{ScanAngle}}$ is a rotation of the observed range into the laser scanner frame by the observed scan angle; and r_{Range} is the range distance determined from the two-way travel time of the laser pulse (El-Sheimy, 2006; Vaughn et al., 1996). The ellipsoidal frame represents the earth-fixed coordinate system and is typically defined with Universal Transverse Mercator (UTM) mapping frame conventions. The ellipsoidal frame axes are orientated toward the north, east, and local ellipsoidal normal, and the GNSS obtains observations within this frame. The local-level frame is oriented with axes pointing toward the north, east, and local geoidal normal, and the inertial measurement unit obtains observations within this frame. The body frame has mutually perpendicular axes toward the front of the aircraft, toward the port side of the aircraft, and down to complete a right-handed system. These two reference frames are reconciled through the roll, pitch, and yaw orientation parameters observed by the inertial measurement unit. The laser scanner frame has axes parallel to the direction of scanning, perpendicular to the laser scanning mirror in its nadir position, and toward the front of the aircraft to complete a right-handed system.

The direct georeferencing equation of lidar is a well-understood and well-used formula. Considerable effort has focussed on ensuring that the combination of observations maintains the highest accuracy possible to ensure final derived point coordinates have minimal error (Vaughn et al., 1996; Baltsavias, 1999; Schenck, 2001). High-accuracy lidar observations became possible after the introduction of differential satellite positioning and advancements in the design of contemporary gyroscopes, originally responsible for the majority of error. Following this, the largest source of error within the direct georeferencing was the proper calculation of the boresight misalignment parameters populating the $\mathbf{R}_{\mathrm{Boresight}}$ matrix. Several studies have been produced, and research continues into the reduction of errors and the errors introduced through these rotations (Morin, 2002; Skaloud and Lichti, 2006). As the direct georeferencing gains in accuracy, initially trivial components begin to gain significance.

Currently, the $\mathbf{R}_{\mathrm{G}}$ matrix containing the rotation of the deflection of the vertical from the local-level reference frame to the ellipsoidal reference frame has gone largely ignored. Vaughn et al. (1996) claim that with current (ca. 1996) inertial measurement equipment this factor was trivial, although its effect could become significant with implementation of higher accuracy inertial measurement equipment. Since the publication of the study of Vaughan et al., inertial technology has experienced significant gains in accuracy. The gyroscope accuracy used by Vaughn et al. was 0.05° (180″) in each of the observed aircraft attitude states. Contemporary inertial measurement units (IMUs), such as the POS AV510 produced by Applanix, claim that accuracies reach a level of 0.005° (18″) in roll and pitch and 0.008° (29″) in heading, indicating an increase in accuracy by an order of magnitude in roll and pitch and by nearly an order of magnitude in heading. Despite these advancements, there has been no report on whether the increased accuracy requires the inclusion of the deflection of the vertical, and this component is often not considered in commercial georeferencing algorithms.

To determine the significance of the deflection of the vertical with respect to observed ground coordinates, the accuracy of commercial lidar systems must be compared against the systematic bias created by ignoring this effect. Optech is the world's largest manufacturer and vendor of lidar systems (TMS International Ltd., 2005). The published vertical accuracy of the Optech ALTM 3100 EA lidar system in the absence of GPS error is 10–20 cm (depending on flying height), and the published horizontal accuracy is equivalent to the flying height divided by 5500 (Ussyshkin and Smith, 2006). This paper aims to determine whether ignoring the deflection of the vertical will cause errors that are significant to these limits. The paper begins by explaining the theoretical background of the deflection of the vertical, with a focus on its significance to lidar observations. Following this, a sensitivity analysis is performed that indicates the significance of the deflection of the vertical to observed coordinates. The error in the determination of the deflection of the vertical is also considered in the lidar error budget.

It should be noted here that the repercussions of the effect of the deflection of the vertical within the hardware IMU system and the position and orientation software algorithms are not considered. It is assumed that the output downward orientation of the IMU subsystem is perfectly aligned with the direction of local gravity. Various errors within the raw IMU measurements, platform misalignment, and errors during the processing of the trajectory may serve to absorb or reduce the bias introduced through ignoring the deflection of the vertical. The analysis provides a preliminary assessment of these errors, with some emphasis on geographic areas in which it may serve to bias lidar observations. To that end, a simple approach has been presented that can be easily adopted by mapping professionals who have access to only the lidar point cloud and processed trajectory. Readers should be aware that accounting for these additional error parameters will potentially increase the accuracy of the presented methodology. This assessment is of considerable value to appropriate quality assurance reporting and provides the context of the potential error magnitudes.

Deflection of the vertical

A discussion relating to deflection of the vertical requires knowledge of several basic geodetic concepts. A detailed review of these is outside the scope of this paper, and the interested reader is referred to Vaníček and Krakiwsky (1986) and Hofmann-Wellenhof and Moritz (2005).

Vaníček and Krakiwsky (1986) succinctly describe the deflection of the vertical as "the angle between the actual plumb line taken on the geoid and the ellipsoidal normal." This definition indicates that the calculation of the deflection of the vertical requires modelling two surfaces, namely the geoid and the ellipsoid. The geoid is a surface with constant gravitational potential. Surfaces with the property of constant potential are termed equipotential surfaces. The geoid is one realization of many possible equipotential surfaces but is uniquely defined to coincide with mean sea level. The surface is realized from a geopotential model and several other auxiliary pieces of information (Smith, 1998). The direction of gravity will always be perpendicular to an equipotential surface.

Since gravitational force is proportional to mass, varying mass distributions throughout the Earth's body cause the undulation of equipotential surfaces to vary. An increase in density within the Earth causes a strengthening in the force of gravity which reduces the separation of equipotential surfaces. Similarly, a decrease in density causes an increased separation between successive equipotential surfaces. The result is that varying mass distributions within the Earth's body cause heterogeneity in equipotential surfaces that is often difficult to model.

The force of gravity is a physical phenomenon that enables terrestrial observations to be referenced to a common surface, usually the geoid. Unfortunately, the heterogeneous structure of equipotential surfaces is difficult to describe mathematically. Positioning for the purpose of georeferencing on the Earth's surface is more easily facilitated with the use of a mathematically simple entity such as an ellipsoid. A global approximation of the geoid is obtained with a geocentric ellipsoid, termed the reference ellipsoid, that has its minor axis orientated parallel to the principle axis of inertia of the Earth (Vaníček and Krakiwsky, 1986). However, due to the varying surface structure and mass distributions within the Earth, the ellipsoidal approximation introduces error. The deviations between the geoid and reference ellipsoid are termed geoidal undulations. **Figure 1** diagrams the geoidal undulations between a realization of the geoid derived from a global geopotential model, the EGM-08 and the GRS-80 reference ellipsoid. Data for **Figure 1** were obtained from the National Geospatial Intelligence Agency (NGA) (http://earth-info.nga.mil/GandG/wgs84/gravitymod/egm2008/index.html).

The undulating structure of the geoid is apparent in **Figure 1**. The most striking variations occur in the South Pacific and south of India. Geoidal undulations also follow smooth transitions, as the gravity field does not experience discontinuities. A close inspection reveals that mountain chains are correlated with more pronounced geoidal undulations due to mountain roots causing changes in the density of the Earth's upper crust.

In addition to the geoidal undulation, the ellipsoidal approximation also causes a variation in the normal direction of the true geoidal surface and ellipsoidal approximation. The difference between the normal vectors is the deflection of the vertical. **Figure 2** illustrates the phenomenon.

Recall that the direct georeferencing requires angular and positional accelerations sensed by three mutually perpendicular accelerometers and three mutually perpendicular gyroscopes. The inertial measurement unit is initially referenced to the sensed direction of gravity by the accelerometers (Jekeli, 2001), and this direction is perpendicular to the geoid. Subsequently, the observed attitude angles (roll, pitch, and yaw) are also referenced to the direction of gravity. However, the GNSS

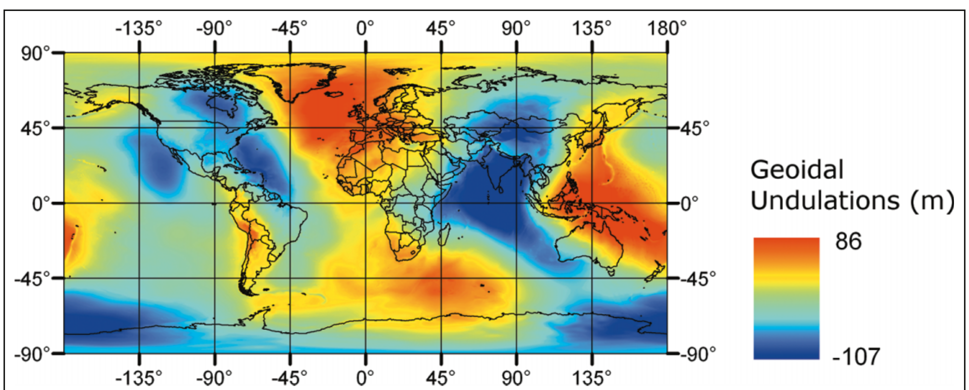

Figure 1. Geoidal undulations realized through the EGM-08 geopotential model. Data obtained from the NGA.

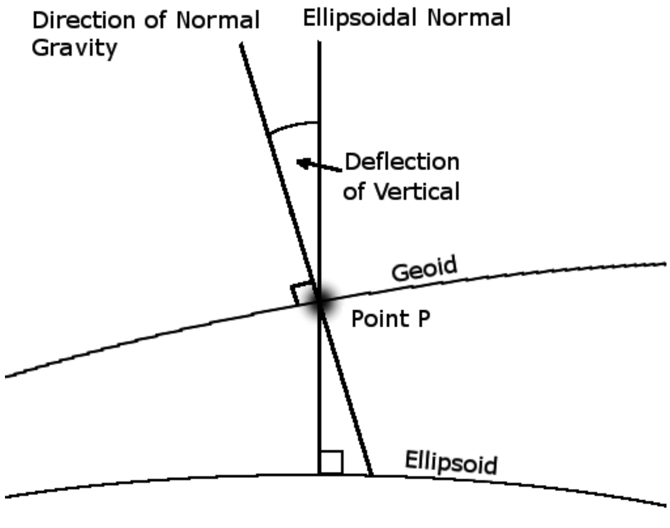

Figure 2. Deflection of the vertical at point P is represented by the angular difference between the normal direction to the ellipsoid and geoid.

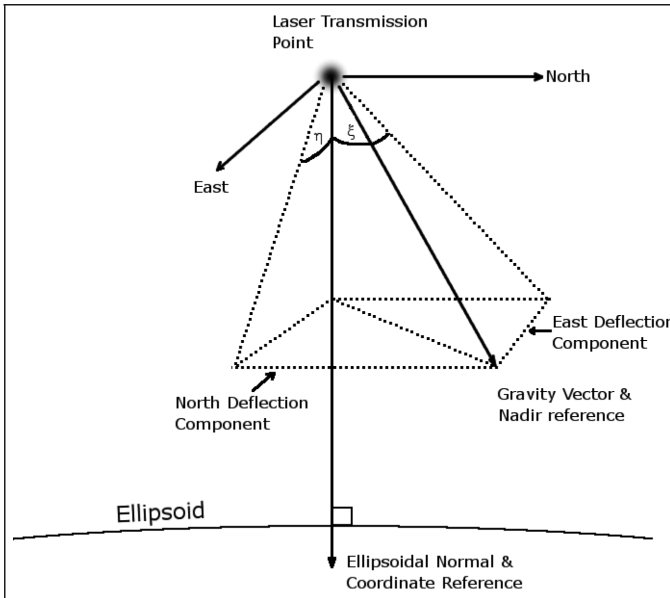

Figure 3. Effect of deflection of the vertical components applied to lidar scenario. The scan angle and IMU observations are referenced to the gravity vector, and the coordinates are referenced to the ellipsoidal normal, resulting in an angular shift of observed features. Adapted from Figure 6.9 of Jekeli (2001).

receiver references the position of the aircraft to the World Geodetic System 1984 (WGS-84) datum, which uses the GRS-80 reference ellipsoid. To properly reconcile the two frames and implement the attitude angles in the ellipsoidal frame, a rotation of the deflection of the vertical is required (Schwarz, 1983; Vaughn et al., 1996). In addition, the scan angle observed by the scanning mirror mechanism is also referenced to the sensed direction of gravity and must also be rotated into the ellipsoidal frame. The deflection of the vertical is typically described by two components, ξ and η, representing the components in the meridian (north–south) and prime vertical planes (east–west) of the reference ellipsoid, respectively (Vaníček and Krakiwsky, 1986). **Figure 2**, adapted from Figure 6.9 of Jekeli (2001), diagrams these circumstances in the georeferencing of lidar observations.

With reference to **Figure 3**, consider that a range observation from a lidar system will be relative to the direction of gravity. However, the direction to the observed ground coordinate is relative to the ellipsoidal normal direction vector. This creates a rotational error equal to the deflection of the vertical. Quantification of the deflection of the vertical can be performed in two ways: (*i*) specialized on-site observations, or (*ii*) analyzing a geopotential model. On-site observations to determine the deflection of the vertical require unique equipment and expertise generally not feasible for lidar campaigns. Considering that in many cases the magnitude of systematic bias introduced by the deflection of the vertical is in close proximity with the inherent noise level in the data, it is much more realistic, efficient, and cost-effective to utilize existing geopotential models.

It can be proven that the Earth's gravitational potential can be represented with a spherical harmonic expansion (Vaníček and Krakiwsky, 1986; Hofmann-Wellenhof and Moritz, 2005; Torge, 2001). Gravity observations must be collected to control a spherical harmonic model and describe the complex variation of the Earth's gravity field. The details of the derivation of the model are not critical for the study presented here, although it is important to realize that the accuracy and spatial resolution of the model are dependent on the number of gravity observations used to control the model. Until recently, the EGM-96 model has been the most accurate and most dense global geopotential model available. Details on its parameters can be obtained from Lemoine et al. (1998). Recently, the EGM-08 (see Pavlis et al., 2008) has been released and supersedes the EGM-96. The accuracy of the EGM-08 geopotential model has improved due to observations that were made available by the Gravity Recovery and Climate Experiment (GRACE), which provided gravity observations in areas that were deficient in the EGM-96 model.

Variable mass distributions in mountainous environments cause unpredictable variations in the gravitational field, and the approximation by a reference ellipsoid suffers. In addition, the accuracy of geopotential models is known to degrade significantly in mountainous environments. The sudden and unpredictable change in the density within the Earth's crust requires numerous gravity observations to properly model the gravitational field. Often measurements of sufficient density are not available. The models tend to perform well in areas of flat homogeneous terrain, as there are very few variations in the gravity field. Therefore, mountainous environments will suffer from the largest magnitudes of the deflection and the most inaccurate determinations. These areas will introduce the largest errors in the georeferencing of lidar observations.

Sensitivity of lidar-derived coordinates to the deflection of the vertical

Knowledge of the theoretical basis of the deflection of the vertical allows us to formulate three postulates that are important to the acquisition of lidar observations: (*i*) deflection of the vertical direction is independent of flying direction; (*ii*) deflections of the vertical experience gradual changes spatially; and (*iii*) deflections are relatively small in magnitude and error in homogeneous terrain and generally large in magnitude and error in heterogeneous terrain. The independence with respect to flying direction ensures that an entire lidar campaign will experience a predictable systematic bias if the deflection of the vertical is ignored. According to the second postulate, the deflection values experience gradual changes spatially, and therefore it is likely that small survey areas will have consistent deflection values and errors. If larger surveys are flown, especially in mountainous areas, local changes in deflection values will cause varying shifts to derived coordinates. To perform the compensation, deflection of the vertical values can be applied to the beam direction of each individual laser pulse. Deflection values determined from the EGM-08 global geopotential model can be obtained from the NGA website mentioned previously.

Considering the magnitude of the deflection of the vertical in the context of targeted error requirements, it is possible that a correction by the deflection of the vertical is unnecessary. The magnitude of the deflection can vary between a few arc-seconds in smooth even terrain and approach two arc-minutes in the most severe circumstances. A correction by a few arc-seconds falls well within the noise level of commercial lidar systems and therefore need not be considered (Goulden and Hopkinson, 2010; Glennie, 2007). As the deflection values increase, however, the significance to the overall lidar error budget also increases. **Figure 4** displays deflection of the vertical magnitudes and direction within Canada as obtained from the NGA. As expected, the effect is generally most severe in the western portion of the country due to the mountainous terrain.

Lidar observation error estimates were simulated by calculating coordinates through Equation (1) twice, once by ignoring the deflection of the vertical and again by including the deflection. The systematic error in ground coordinates can be obtained by differencing the two results. The simulation through the direct georeferencing equation included three separate flying heights above ground, namely 1000, 2000, and 3000 m, as these represent the typical range of altitudes used in operational lidar mapping campaigns. To simplify the direct georeferencing equation, the aircraft was assumed to be experiencing no attitude rotations, and boresight misalignments were assumed to be negligible. This simplification eases the mathematical complexity of the direct georeferencing equation and will result in optimistic error estimates. The objective of the analysis was to focus only on the error contributed by the deflection of the vertical while eliminating influences that could propagate larger errors into the results.

When introducing the deflection of the vertical to the direct georeferencing equation, its direction was assumed to be coincident with the scan angle direction. The simulation was performed in this manner to introduce the maximum vertical error. When the deflection of the vertical direction is perpendicular to the scan angle direction, the resulting vertical error will be negligible. Therefore, the induced vertical error will range between a maximum value when the scan angle direction is parallel to the deflection of the vertical direction and a minimum value when the deflection direction is perpendicular to the scan angle direction. The simulated error scenario is presented in **Figure 5**. Notice that the resulting horizontal error is identical in magnitude

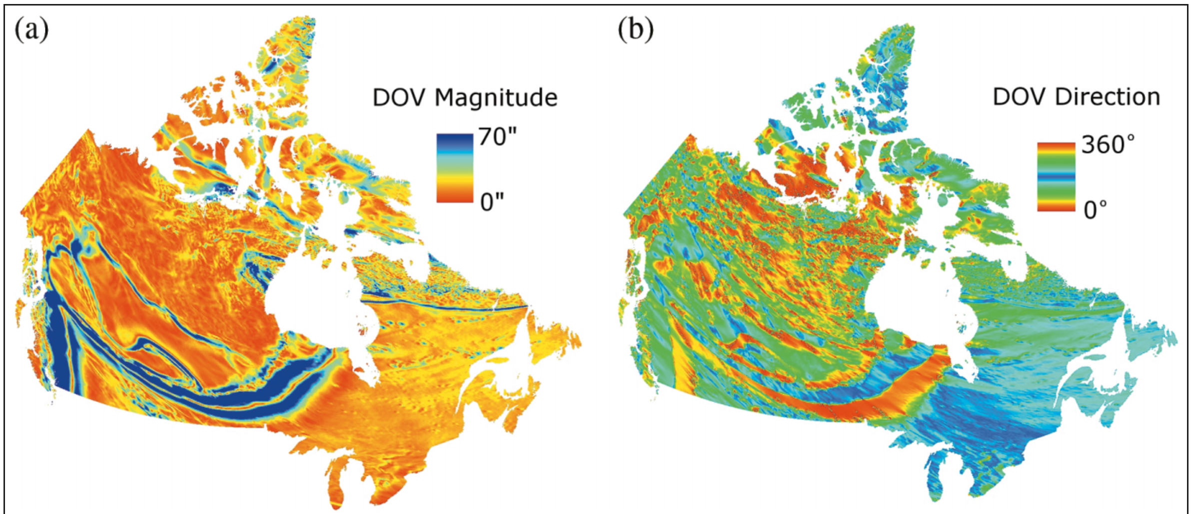

Figure 4. Deflection of the vertical magnitude (a) and direction (b) for the Canadian land mass. Data obtained from the NGA.

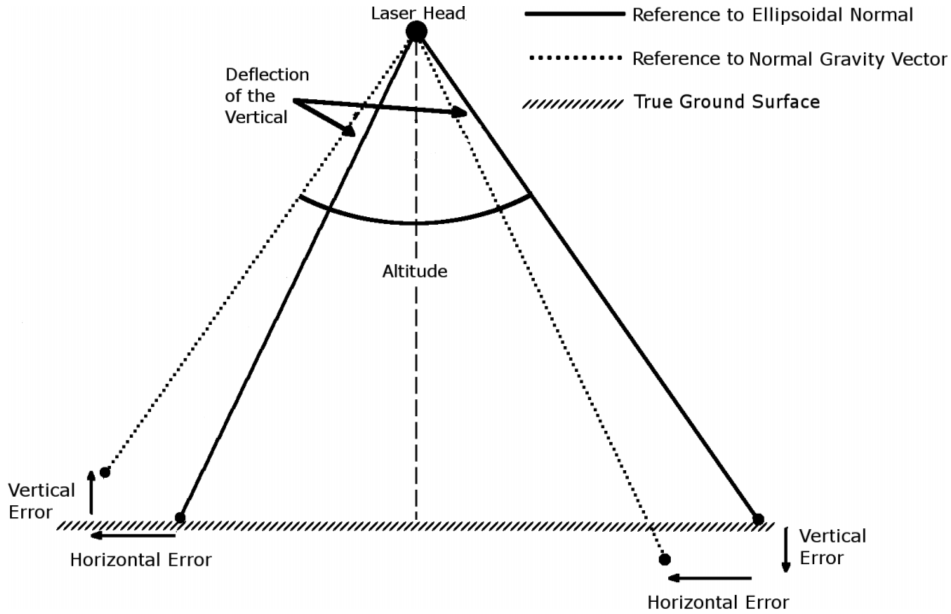

Figure 5. Simulated error induced by ignoring the deflection of the vertical when direction is coincident with scan angle direction. The resulting horizontal error contains similar magnitude and direction, and the vertical error similar magnitude but opposite direction.

and sign, whereas the vertical error is equal in magnitude but opposite in sign. This creates swath edges that are vertically separated by twice the induced vertical error.

The $\mathbf{R}_G$ matrix in the direct georeferencing equation can be determined by combining well-known rotation matrices around the x axis (east–west) and y axis (north–south) of the ellipsoidal mapping frame. The magnitudes of the rotations are defined as the ξ and η components of the deflection of the vertical, and the resulting $\mathbf{R}_G$ is formalized as follows:

$$\mathbf{R}_G = R_y(\eta)R_x(\xi) = \begin{bmatrix} \cos(\eta) & \sin(\xi)\sin(\eta) & \sin(\eta)\cos(\xi) \\ 0 & \cos(\xi) & -\sin(\xi) \\ -\sin(\eta) & \cos(\eta)\sin(\xi) & \cos(\xi)\cos(\eta) \end{bmatrix} \tag{2}$$

The difference between the two calculations of coordinates against the deflection of the vertical is presented in **Figures 6** and **7** as horizontal and vertical errors, respectively. The data provided by the NGA for the EGM-08 geopotential model indicated that the largest magnitude of the deflection of the vertical occurring in Canada is approximately 70 arc-seconds, but the graphs continue slightly beyond this point to describe potentially larger instances in other parts of the world.

The horizontal broken lines in **Figures 6** and **7** indicate the published accuracy for each flying height, and the vertical broken lines correlate the respective deflection of the vertical magnitude that consumes the entire published error budget. No connection is given in the literature for the ALTM 3100 EA system between vertical error and flying height, and therefore it is assumed that the highest accuracy value of 5 cm corresponds to 1000 m, the lowest accuracy value of 20 cm corresponds to an altitude of 3000 m, and the accuracy for 2000 m falls mid-range.

A scan angle of 25° was chosen for the calculation of the vertical error to represent the most pessimistic scenario of controlled survey configurations, as it is the operational limit of the ALTM 3100 EA. The effect of the scan angle on the horizontal error is negligible within the operational limits of typical lidar systems, and therefore no scan angle has been specified. Recall that vertical error can manifest as either a positive or a negative value depending on the scan angle direction. For illustrative purposes, only the absolute error magnitude is shown in **Figure 7**.

The vertical error budget is completely consumed by ignoring the deflection of the vertical at magnitudes of approximately 53″, 40″, and 35″ for flying heights of 1000, 2000, and 3000 m, respectively. The horizontal error budget is completely consumed at deflection of the vertical magnitudes of 34″, 35″, and 37″ for flying heights of 1000, 2000, and 3000 m, respectively. The largest value of the deflection of the vertical in Canada is approximately 70″, and it is not uncommon for areas within the Rocky Mountains to reach well above 40″. This indicates that the deflection of the vertical component is significant in the direct georeferencing calculations in these scenarios. For the purposes of quality assurance, an error component is generally considered significant if it reaches 10%–33% of the total error budget. The error due to ignoring the deflection of the vertical will reach significance in this context at much lower magnitudes. Users should be aware of the limitations in accuracy in various parts of the country. If contracted accuracy requirements for the lidar survey or quality of final derived products are

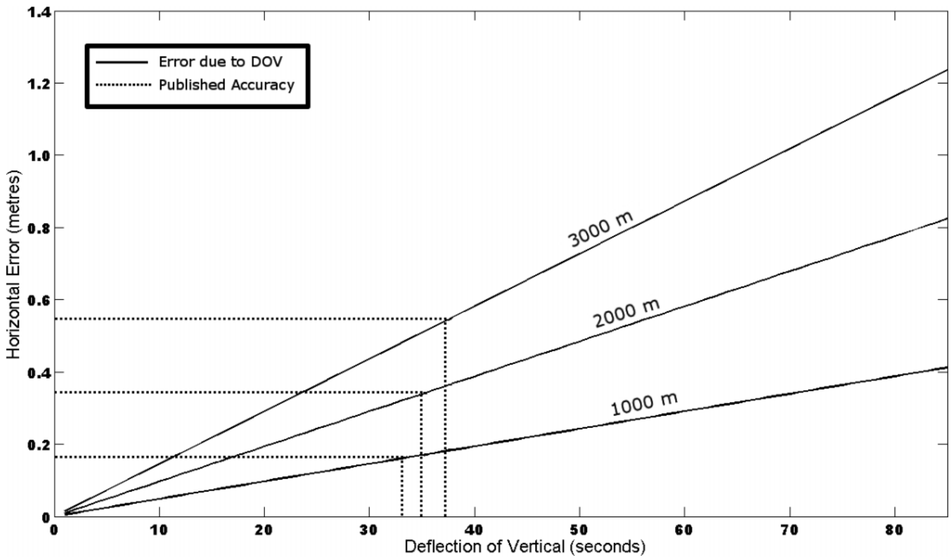

Figure 6. Horizontal error caused by ignoring the deflection of the vertical. Errors are displayed with separate flight altitudes of 1000, 2000, and 3000 m. Published accuracy is represented by the horizontal broken lines and correlated with deflection of the vertical magnitude.

going to suffer as a result of ignoring this component, then users might wish to consider applying corrections.

Since ignoring the deflection of the vertical causes consistent horizontal shifts in the dataset regardless of flight direction and scan angle, it is tempting to match the lidar dataset with independent ground validation and shift the dataset accordingly to coincide. Although this technique "may" prove successful and is easily performed, it is not considered appropriate. As stated in section A.8 (Airborne light detection and ranging (lidar) surveys) of the US Federal Emergency Management Agency (FEMA, 2003, p. A52) *Guidelines and specifications for flood hazard mapping partners*, "It is relatively easy to determine the magnitude of systematic errors and adjust all data accordingly; however, the assigned mapping partner must not "correct" such errors until the source is clearly identified and documented." Therefore, prior to correction of the error caused by the deflection of the vertical, the appropriate value should be obtained and the data shifted appropriately through the introduction of the value to the direct georeferencing equation.

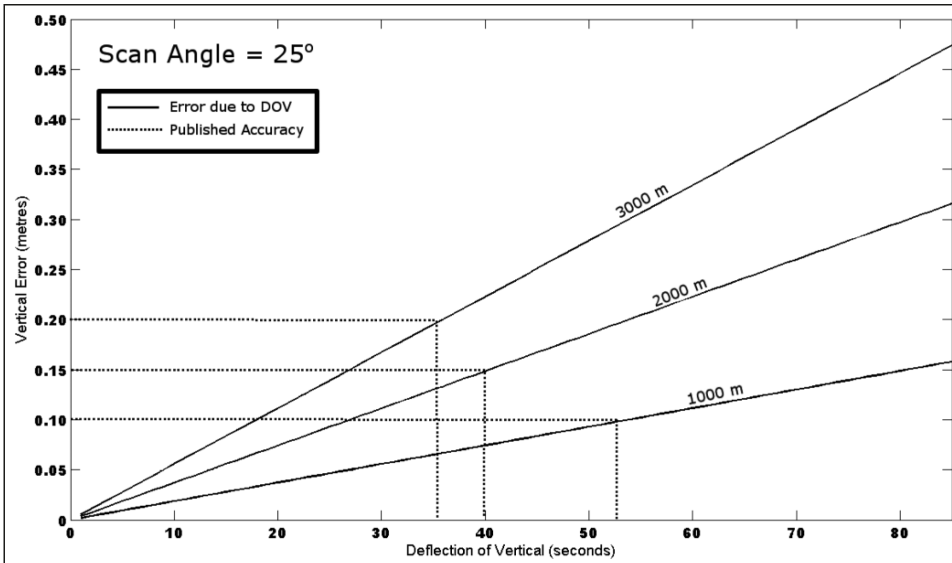

Figure 7. Absolute vertical error caused by ignoring the deflection of the vertical when the direction is coincident with scan angle direction. Errors are displayed for separate altitudes of 1000, 2000, and 3000 m. Published accuracy for each altitude is represented by the horizontal broken lines and correlated with deflection of the vertical magnitude.

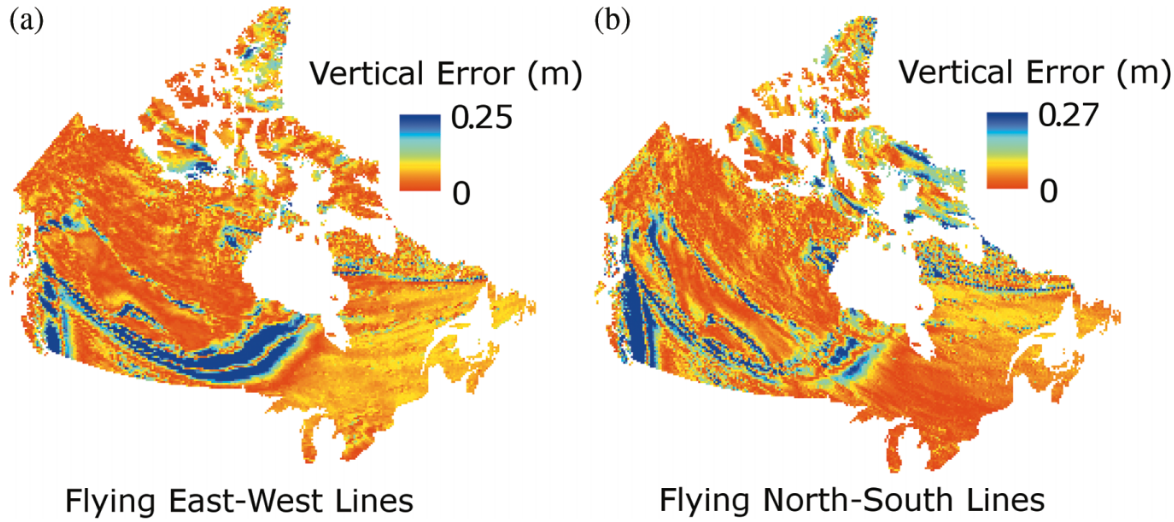

Figure 8. Induced absolute vertical error to lidar-derived coordinates within Canada due to ignoring the deflection of the vertical and flying east–west lines (a) and north–south lines (b). Areas represented in red represent no induced error due to negligible deflection values, and areas in blue represent the highest instances of error. Altitude 200 m and scan angle 25°.

To illustrate the difference in the absolute vertical error introduced as a function of flying direction, the error was mapped within Canada according to the deflection magnitudes shown in **Figure 8**. A flying height of 2000 m and scan angle perpendicular to the flight direction of 25° were chosen for the analysis. The induced vertical errors for flight lines in the north–south and east–west directions are shown. By comparing with the deflection of the vertical direction map in **Figure 4**, it is evident that flying east–west lines results in larger errors when the direction of the deflection of the vertical is north–south, and vice versa. Therefore, with knowledge of the direction of the deflection of the vertical, lidar survey flight line orientation could potentially be planned in such a way as to mitigate this vertical error component.

Errors in the determination of the deflection of the vertical

If lidar observations are going to be corrected by the deflection components, it is prudent to identify the level of error that exists in the correction for proper quality assurance reporting. The most accurate, accessible, and studied geopotential model currently publically available is the EGM-96. Although the EGM-08 model is more accurate, the time elapsed since its availability has not allowed error statistics to be generated and published. Therefore, the EGM-96 model is used as a basis for error analysis.

In a plot of the error along a profile of the continental United States presented by Jekeli (2001), the EGM-96 model performed well in the eastern United States where terrain is relatively flat. In the western United States, which contains a large portion of the Rocky Mountains chain, there were deviations of over 10″. In Table 2 reported in Jekeli (1999), maximum deflection of the vertical errors in the northwest were 20–22″, whereas maximum differences in the northeast and southeast were 11″. The root mean square (RMS) errors in the western United States were over 6″ in the north and 7″ in the south, and the error in the model was approximately 4–5″ in the eastern regions of the country (Jekeli, 1999).

The worst-case scenario of North America is tested to identify how these errors might affect lidar observations. This situation occurs in highly mountainous environments in western Canada or the United States. Since the maximum error noted by Jekeli (1999) was 22″, this is used as the test case. Similar to the bias caused by ignoring the deflection of the vertical, errors in the deflection of the vertical are purposely introduced into the direct georeferencing equation to determine the bias in final point coordinates. The instance of error in the determinations differs from the correction of the deflection of the vertical in that it can be considered a random source of error constituting an unknown magnitude and direction. Although it is possible that the error in the deflection reaches magnitudes similar to that of the deflection itself, this is unlikely because the largest incidence of error is typically in areas where the magnitude of the deflection is also large. This should not discourage the correction of the known systematic influence of the deflection of the vertical discussed previously because the overall accuracy will still be increased.

The maximum error of 22″ was introduced into a variety of survey configurations to observe the bias in error under different circumstances. Test cases were split into both horizontal and vertical results. The survey settings that were modified during testing were the altitude and scan angle. The altitude was chosen to be 1000, 2000, and 3000 m for consistency with the results presented in **Figures 6** and **7**. The results can be obtained from **Figures 6** and **7** by accessing the

Table 1. Horizontal error induced by a 22″ error in the determination of the deflection of the vertical.

Altitude (m)	Horizontal error (m)
1000	0.11
2000	0.21
3000	0.32

error level corresponding to the 22″ deflection of the vertical value on the horizontal axis. These values for the horizontal error are reported in **Table 1**. Since the vertical error is dependent on scan angle, the results are presented in **Figure 9** with scan angles ranging from 0–25°.

The values displayed in **Table 1** and **Figure 9** suggest that for surveys performed in mountainous terrain the error component of the deflection of the vertical has the capability of being responsible for a significant portion (>10%) of the total error which cannot be accounted for unless specific observations are taken within the survey area to determine a more accurate value for the deflection of the vertical. It is prudent to note that this error level is of similar magnitude to the noise level of the roll and pitch attitude determinations (~18″) and that these errors will manifest differently in the lidar point cloud. The error due to the deflection of the vertical is random in the sense that it constitutes an a priori unknown direction and magnitude; however, it is also systematic in that the error is likely to be consistent over small survey areas. Essentially, the error will be additive to the bias introduced by ignoring the deflection of the vertical. Errors in attitude determination will generally be randomly distributed throughout the point cloud and will not create an identifiable systematic bias. Although the error magnitudes are similar, the eventual consequence to derived lidar points will be different and worth separate treatment in quality assurance reporting. However, it is possible that the bias introduced by the error in the deflection of the vertical will be lost within the error introduced by the attitude determination and will not require explicit consideration.

Considering that published error budgets for lidar surveys do not specify terrain conditions, it is likely that the published values are optimistic for surveys in mountainous terrain. Although inaccurate measurements in mountainous terrain can be attributed to a number of other factors such as surface slope (Hodgson and Bresnahan, 2004), the error due to the deflection of the vertical will serve to further degrade overall accuracy. However, unlike effects due to slope, this factor can be predicted and eliminated if dense local geopotential models or terrestrial observations are used to determine a more accurate value for the deflection of the vertical and are included in postprocessing.

Conclusion

The direct georeferencing equation of lidar contains the combination of observations from many sources. One such observation is the deflection of the vertical component represented by $\mathbf{R}_G$ as shown in Equation (1). Since on-site observations to determine the deflection of the vertical are generally not performed in concert with lidar surveys, the value for this component is more easily obtained from a geopotential model. This component was correctly assumed to be negligible during the early development of lidar technology. However, with increased accuracy in inertial motion observations, improved calibration procedures, and expanded operational envelopes leading to survey capabilities in high mountainous terrains, its proportional contribution to the overall error budget has increased.

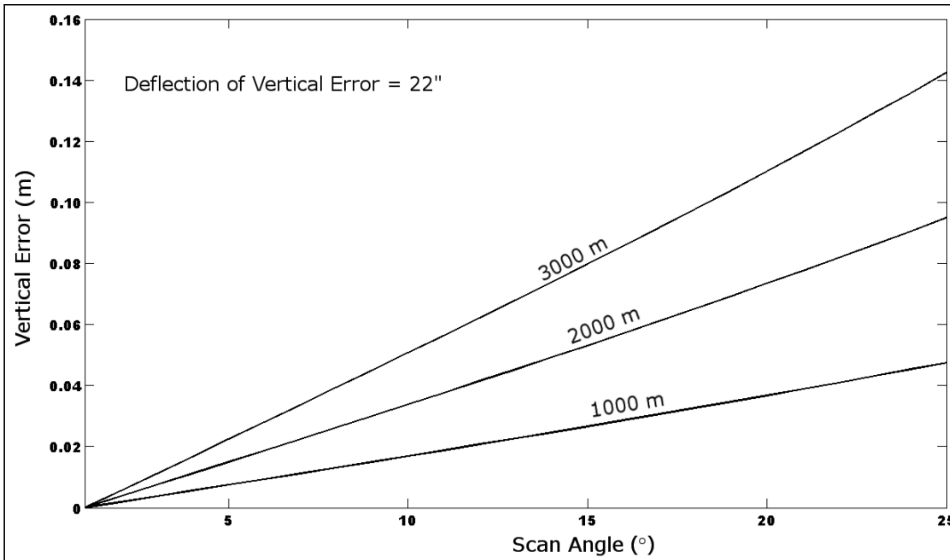

Figure 9. Simulated change in vertical errors by varying altitude (1000, 2000, and 3000 m) and scan angle (0–25°) by introducing an error in the deflection of the vertical of 22″.

Ignoring the deflection of the vertical in direct georeferencing calculations can completely consume the published horizontal error budget of the ALTM 3100 EA lidar systems at magnitudes of 34″, 35″, and 37″ for flying heights of 1000, 2000, and 3000 m, respectively. The published vertical error budget can be consumed at deflection of the vertical values of 53″, 40″, and 35″ for flying heights of 1000, 2000, and 3000 m, respectively, and a scan angle of 25°. Large deflection of the vertical values occurs in mountainous environments such as those in western Canada. According to the most recent publically available global geopotential model, namely the EGM-08, the largest value of the deflection of the vertical in Canada is 70″. This is well above the threshold for consuming the published lidar error budget, and much of western Canada is characterized by similar deflection of the vertical values. User striving to meet stringent accuracy requirements should be aware that current commercial lidar algorithms typically do not consider this component, and therefore data will have to be corrected in postprocessing. This can be done by accessing deflection values for the survey area from the EGM-08 geopotential model and applying the deflection of the vertical to the beam direction of each individual laser pulse.

The deflection of the vertical will cause a predictable error in derived coordinates. Horizontal errors will be independent of flight direction, scan angle direction, and scan angle magnitude, whereas vertical errors are dependent on each of these factors. If the deflection of the vertical direction is perpendicular to the scan angle direction, the introduced vertical errors will be negligible. Maximum vertical errors occur when the deflection of the vertical direction is parallel to the scan angle direction and the scan angle magnitude is at a maximum. Lidar flights can potentially be planned to reduce the induced vertical error by orienting flight lines parallel to the direction of the deflection of the vertical.

Using a geopotential model for derivation of the deflection of the vertical will also contain error due to the observations used to produce the model and its inability to properly model the gravitational field. Proper quality assurance of lidar observations should report on the possible errors in this component. Model errors tend to coincide with large deflection magnitudes and therefore are also prevalent in mountainous environments such as the western region of the North American continent. Existing error analyses of the EGM-96 model yielded maximum errors of 22″ in the western United States. If this value is systematically inserted into the lidar direct georeferencing equation, the derived coordinates contain up to 32 cm of error horizontally and 15 cm vertically at high altitudes and large scan angles. This error cannot be avoided without the integration of higher accuracy values for the deflection of the vertical, and therefore its existence should be accepted and reported in quality assurance procedures. In surveys that are being flown in environments known to contain large errors in the deflection of the vertical, the required error budget for the survey should be considered to determine whether more accurate values for the deflection of the vertical need to be obtained, or a flight pattern should be considered that would render the vertical error negligible. Future work in this area will require focus on analyzing the systematic errors within the IMU hardware system and the processing routines used to produce the aircraft trajectory to determine its effect on the correction algorithm presented here. In addition, lidar observations in areas with large deflections of the vertical will have to be collected and compared with independent validation data to determine the success of the proposed methodology.

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