Exercise 1.3
Logarithms

What is a logarithm?

$$\log_a(a^x) \equiv x$$

the "base" of the logarithm

Essentially, taking the logarithm of a number is asking yourself the question: "If this number was expressed as "base" to an exponent, what would the exponent be?" Thus, a logarithm is the inverse function of an exponential.

e.g. If "base" = 10, we can say that $\log_{10}(1000) = \log_{10}(10^3) = 3$

In CHEM 1000 and 2000, if no base is shown, assume base 10.

$$\log(x) \equiv \log_{10}(x)$$

In CHEM 1000 and 2000, ln ("natural logarithm") is used for logarithms that are base e.

e is "Napier's number" and has the value 2.718281828….

$$\ln(x) \equiv \log_e(x)$$

How can logarithms be used to "pull down" a variable from an exponent?

Take the logarithm of both sides of the equation. The base of your logarithm must be the same as the base of your exponent.

e.g. To solve for x in $10^x = 1000000$, take the base 10 logarithm of both sides of the equation:

$$10^x = 1000000$$
$$log_{10}(10^x) = log_{10}(1000000)$$
$$x = log_{10}(1000000)$$
$$x = 6$$

How can we "pull a variable out of" a logarithm?

Given that a logarithm is the inverse function of an exponential, we can also say that:

$$a^{\log_a(x)} \equiv x$$

e.g. If "base" = 10, we can say that $10^{\log_{10}(1000)} = 10^3 = 1000$

Therefore, to solve for a variable within a logarithm, make both sides of the equation into exponents where the base is the same as the base of the logarithm.

e.g. To solve for x in $log_{10}(x) = 2$, take 10 to the power of each side of the equation:

$$log_{10}(x) = 2$$
$$10^{log_{10}(x)} = 10^2$$
$$x = 10^2$$
$$x = 100$$

Useful exponent relationships:

$$a^0 = 1$$

unless a = 0
but that's for math class… ☺

$$\frac{1}{a^x} = a^{-x}$$

$$a^x \cdot a^y = a^{x+y}$$

$$\frac{a^x}{a^y} = a^{x-y}$$

$$(a^x)^y = a^{x \cdot y}$$

$$a^{\left(\frac{1}{x}\right)} = \sqrt[x]{a}$$

$$a^x \cdot b^x = (ab)^x$$

Useful logarithmic relationships:

$$log_a(1) = 0$$

$$log_a\left(\frac{1}{x}\right) = -log_a(x)$$

$$log_a(x \cdot y) = log_a(x) + log_a(y)$$

$$log_a\left(\frac{x}{y}\right) = log_a(x) - log_a(y)$$

$$log_a(x^y) = y \cdot log_a(x)$$

$$log_b(x) = \frac{log_a(x)}{log_a(b)}$$

therefore $ln(x) = \frac{log(x)}{log(e)}$

1. Solve for x in each of the following equations WITHOUT USING A CALCULATOR.

(a) $x = log(10)$ (b) $x = log(0.01)$ (c) $x = log\left(\frac{1}{1000}\right)$

2. Simplify each of the following expressions as much as possible.

(a) $log(10^{-pH})$ (b) $log(10x^2)$ (c) $e^{ln(x)}$

3. Solve for x in each of the following equations.

(a) $13.2 = -log(x)$

(b) $13.2 = -ln(x)$

(c) $ln\left(\frac{x}{5}\right) = 1.50$

(d) $ln\left(\frac{x}{5}\right) = -1.50$

(e) $13.2 = e^{x}$

(f) $13.2 = e^{x^2}$