Exercise 2.4
Energy of Nuclear Reactions

Calculating the Energy Change for a Nuclear Reaction

In any nuclear reaction, the exact masses of the products and reactants differ. If the products have a smaller total mass than the reactants, energy is released. The convention is to subtract the mass of the reactants from the mass of the products such that a negative value for Δm corresponds to a *mass defect*:

$$\Delta m = \sum m_{products} - \sum m_{reactants}$$

Einstein's equation describes the relationship between energy and mass. As such, the mass defect can be used to calculate the energy change for a nuclear reaction. A negative Δm results in a negative ΔE and a release of energy:

$$\Delta E = \Delta m c^2$$

This means that in order to calculate the energy change for a nuclear reaction, you must first have a balanced nuclear equation so that you include all products and reactants *<u>except electrons and positrons</u> (see below).*

Why We Don't Include Electrons or Positrons When Calculating the Energy Change for a Nuclear Reaction *(courtesy of Prof. Marc Roussel)*

…in Beta Emission Reactions (Beta Decay)

The first thing to be aware of is that, when we balance nuclear reactions, we focus on the nuclei. We don't think at all about what happens to the electrons in these processes. So let's properly account for those electrons in an example of a beta emission reaction and see what this tells us. For the beta decay of ^{99}Mo, we would write the following nuclear reaction:

$$^{99}_{42}\text{Mo} \rightarrow {}^{99m}_{43}\text{Tc} + {}^{0}_{-1}\beta$$

(The *m* in ^{99m}Tc stands for "metastable". It designates a particular nuclear state. It's not relevant to balancing, so just ignore it.) A neutral molybdenum atom has 42 electrons to balance the nuclear charge of +42. A neutral Tc atom, whose mass would be given in a table of isotopic masses, would have *43* electrons. That's **<u>not</u>** what we make in the nuclear decay process, which releases a negatively charged beta particle (i.e. an electron). What we actually make is a Tc^+ ion, with 42 electrons and 43 protons. So, really, we should write:

$$^{99}_{42}\text{Mo} \rightarrow {}^{99m}_{43}\text{Tc}^+ + {}^{0}_{-1}\beta$$

The mass of the $^{99m}\text{Tc}^+$ ion would be the mass of a ^{99m}Tc atom minus the mass of an electron (and a small correction for the binding energy of that electron, but that's really small compared to the mass defect due to the nuclear process). Δm for this beta decay is therefore really:

$$\Delta m = \left[m\left({}^{99m}\text{Tc}\right) - m_e\right] + m_e - m\left({}^{99}\text{Mo}\right)$$

The quantity in the square brackets is the mass of the $^{99m}\text{Tc}^+$ ion. You can see that the mass of the electron cancels, and we're left with:

$$\Delta m = m\left({}^{99m}\text{Tc}\right) - m\left({}^{99}\text{Mo}\right)$$

…in Electron Capture Reactions

Let's now think about an electron capture process. Potassium-40 decays by electron capture:

$$^{40}_{19}\text{K} + {}^{0}_{-1}\text{e} \rightarrow {}^{40}_{18}\text{Ar}$$

Again, think about the electrons. A neutral atom of potassium has 19 electrons; a neutral atom of argon has 18 electrons. On the surface, it looks like there are 20 electrons on the left-hand side of this equation (the 19 in the potassium atom, plus the electron that will be captured) but that's not what we're trying to represent here. What we're trying to show is that a potassium-40 nucleus captures an electron. That electron is one of the electrons in the atomic orbitals of the potassium atom. So really, the reaction is just:

$$^{40}_{19}\text{K} \rightarrow {}^{40}_{18}\text{Ar}$$

The equation for Δm is obvious once you write the reaction this way. So why don't we? Because the nuclear balancing method allows us to connect reactant nuclei to product nuclei, with particles that are captured or emitted to effect the nuclear changes explicitly shown.

…in Positron Emission Reactions

The last case to consider is positron emission. Phosphorus-30 decays by positron emission:

$$^{30}_{15}\text{P} \rightarrow {}^{30}_{14}\text{Si} + {}^{0}_{1}\beta$$

A neutral atom of phosphorus has 15 electrons, and those 15 electrons remain after the reaction, so we actually make a Si^- ion given that the product has 14 protons and 15 electrons. This case seems worse than the other ones: the mass on the right-hand side is the mass of the ^{30}Si atom, plus the extra electron it carries, plus the positron. So how can we ignore the electron and positron masses? The assumption is that the positron, being the antiparticle of the electron, will meet an electron somewhere and annihilate it:

$$\beta^+ + \text{e}^- \rightarrow \text{energy}$$

It won't be the extra electron carried by the silicon atom, but overall, we'll have destroyed an electron (somewhere) with our positron. The energy of that annihilation is included in the overall Δm calculation when we leave out the masses of the electron carried by the silicon-30 atom and of the positron.

As a side note, if you think about these processes and the fact that beta particles and positrons tend to escape from their respective radioactive materials, you will realize that beta emission and positron emission tend to leave behind a charged sample. In fact, one of the characteristics of highly radioactive beta or positron emitters is that they tend to build up large static charges.

1. One of the many different fusion reactions occurring in stars such as our sun is "carbon burning" – the fusion of two ^{12}C to give ^{20}Ne (emitting ^{4}He). Calculate the energy released by this reaction.

(a) Report your answer in J/mol. *(This refers to J per mole of ^{20}Ne produced.)*

(b) Report your answer in J. *(This refers to J per molecule of ^{20}Ne produced.)*

2. ^{26}Mg is produced when ^{26}Al undergoes electron capture.

(a) Write a balanced equation for this nuclear reaction.

(b) Count the electrons on each side of your balanced equation. It should appear that one of them have "gone missing"! What happened to it? What effect will this have on how we calculate the energy released by this reaction?

(c) Calculate the energy released by this reaction. Report your answer in J/mol and in J

3. While many different fusion reactions occur in stars such as our sun, the main one is the proton-proton chain reaction. In this reaction, four ^{1}H combine (in multiple steps) to give ^{4}He, two positrons, two neutrinos and gamma radiation.

(a) Write a balanced equation for this nuclear reaction.
You do not need to include the neutrinos or the gamma radiation in your equation.

(b) Count the electrons in the atoms on each side of your balanced equation. It should appear that two of them have "gone missing"! What happened to them? What effect will this have on how we calculate the energy released by this reaction?

(c) Calculate the energy released by this reaction. Report your answer in J/mol and in J.

$$M_{{}^{0}_{+1}\beta} = 0.000\ 548\ 580\ \text{u}$$
$$M_{{}^{0}_{-1}\beta} = 0.000\ 548\ 580\ \text{u}$$
$$M_{{}^{1}_{1}H} = 1.007\ 825\ 032\ \text{u}$$
$$M_{{}^{4}_{2}He} = 4.002\ 603\ 254\ \text{u}$$
$$M_{{}^{12}_{6}C} = 12.000\ 000\ 000\ \text{u}$$
$$M_{{}^{20}_{10}Ne} = 19.992\ 440\ 176\ \text{u}$$
$$M_{{}^{26}_{12}Mg} = 25.982\ 592\ 968\ \text{u}$$
$$M_{{}^{26}_{13}Al} = 25.986\ 891\ 904\ \text{u}$$