Answers to Exercise 4.1
Heisenberg and deBroglie

1.

(a) **Step 1: Calculate the mass of an electron in kg**

$$m = 5.485799 \times 10^{-4}u \times \frac{1.660539\times10^{-27}kg}{1u} = 9.109383 \times 10^{-31}kg$$

Step 2: Calculate the deBroglie wavelength of the electron

$$\lambda = \frac{h}{\rho} \qquad\qquad \textit{and} \qquad\qquad \rho = mv$$

$$\lambda = \frac{h}{mv} = \frac{\left(6.626070\times10^{-34}\frac{J}{Hz}\right)}{(9.109383\times10^{-31}kg)\left(2.2\times10^{6}\frac{m}{s}\right)} \times \frac{1Hz}{1\frac{1}{s}} \times \frac{1\frac{kg\cdot m^2}{s^2}}{1J} = 3.3 \times 10^{-10}m$$

$$\lambda = 3.3 \times 10^{-10}m \times \frac{10^{12}pm}{1m} = 3.3 \times 10^{2}pm$$

Step 3: Check your work

Does your answer seem reasonable? Are sig. fig. correct?

(b) The wavelength of the electron (330 pm) is equal to the circumference of the ground state hydrogen atom ($2\pi r = 2\pi(53pm) = 333pm = 3.3 \times 10^{2}pm$). An electron (if treated as a particle) is much smaller than the whole atom; the wavelength of the electron must therefore be significantly larger than the size of the electron.

(c) This suggests that it is reasonable for us to expect that an electron in a ground state hydrogen atom behaves like a wave. We can extrapolate that it is likely that electrons in all atoms behave like waves.

If the deBroglie wavelength had been significantly shorter than the size of the electron, we would instead have concluded that the electron behaved primarily as a particle (i.e. that it did not exhibit significant wave-like properties).

2.

(a) **Step 1: Calculate the absolute uncertainty for the electron's speed**

$$\Delta v = (10\%)\left(2.2 \times 10^6 \frac{m}{s}\right) = 2.2 \times 10^5 \frac{m}{s}$$

Step 2: Calculate the uncertainty in the electron's position (assuming no uncertainty in the mass of the electron; mass of electron was calculated in question 1(a))

$$\Delta x \cdot \Delta\rho > \frac{h}{4\pi} \qquad and \qquad \rho = mv$$

$$\Delta x \cdot \Delta(mv) > \frac{h}{4\pi}$$

$$\Delta x \cdot m\Delta v > \frac{h}{4\pi}$$

$$\Delta x > \frac{h}{4\pi m\Delta v}$$

$$\Delta x > \frac{\left(6.626070\times10^{-34}\frac{J}{Hz}\right)}{4\pi(9.109383\times10^{-31}kg)\left(2.2\times10^{5}\frac{m}{s}\right)} \times \frac{1Hz}{1\frac{1}{s}} \times \frac{1\frac{kg\cdot m^2}{s^2}}{1J}$$

$$\Delta x > 2.6 \times 10^{-10}m \times \frac{10^{12}pm}{1m}$$

$$\Delta x > 2.6 \times 10^{2}pm$$

Step 3: Check your work

Does your answer seem reasonable? Are sig. fig. correct?

(b) **Step 1: Calculate the absolute uncertainty for the electron's speed**

$$\Delta v = (1\%)\left(2.2 \times 10^6 \frac{m}{s}\right) = 2.2 \times 10^4 \frac{m}{s}$$

Step 2: Calculate the uncertainty in the electron's position (same formula derivation as in question 2(a))

$$\Delta x > \frac{h}{4\pi m\Delta v}$$

$$\Delta x > \frac{\left(6.626070\times10^{-34}\frac{J}{Hz}\right)}{4\pi(9.109383\times10^{-31}kg)\left(2.2\times10^{4}\frac{m}{s}\right)} \times \frac{1Hz}{1\frac{1}{s}} \times \frac{1\frac{kg\cdot m^2}{s^2}}{1J}$$

$$\Delta x > 2.6 \times 10^{-9}m \times \frac{10^{9}nm}{1m}$$

$\Delta x > 2.6nm$ *(approximately 2600 pm)*

Step 3: Check your work

Does your answer seem reasonable? Are sig. fig. correct?

(c) To have no uncertainty in our knowledge of the electron's position, we would have to have greater than infinite uncertainty about its momentum (and therefore about its speed). In other words, we would have to have less than no information about the electron's momentum – which is impossible.

(d) This suggests that an electron cannot possibly have a regular orbit around the nucleus. If it did, we would be able to accurately predict both position and momentum after enough measurements.