

Bell's Theorem and the “Bell Telephone”— Where We Stand Today

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ABSTRACT

In 1964 the Irish physicist John S. Bell made what has been called “the most profound discovery of science.” He proved mathematically that the predictions of quantum mechanics for “entangled” systems violate locality, which (roughly speaking) is the view that no physical influence can propagate faster than the vacuum speed of light. This statement is called Bell’s Theorem. By the early 1980s experimental work by Alain Aspect and several others had confirmed Bell’s Theorem to high precision. The debate about the meaning of this result still continues. I will review the orthodox interpretation of Bell’s Theorem, according to which quantum mechanics and the theory of relativity stand in a relation of “peaceful coexistence,” and a minority heretical view (to which I ascribe!) according to which the arguments for peaceful coexistence are flawed. I will conclude by mentioning some recent theoretical work by Y. Aharonov and others that tends to support the heretical view.

Historical Perspective

Main events:

- 1926–27: modern quantum mechanics is developed by Heisenberg, Schrödinger, Born, Dirac, and many others.
- 1927: Heisenberg enunciates the uncertainty relations: if P , Q are non-commuting observables, they obey an uncertainty relation $\Delta P \Delta Q \geq \hbar/2$.

Einstein's (closely related) worries:

- He feels that quantum physics should have a deterministic basis (“God does not play dice with the Universe”) and he is convinced that dispersions such as ΔP are merely statistical uncertainties; particles really must have definite values of P and Q all of the time.
- He is profoundly troubled by “spooky action at a distance”—the fact that quanta are often *more* correlated at a distance than they ought to be on any decent classical view.

Einstein fights a rear-guard action:

- Around 1927 Einstein devises a very detailed causal wave mechanics, hoping to get rid of the probabilities and uncertainties in the theory; however, he finds that his theory is nonlocal and he refuses to publish it; even he cannot make the nonlocality go away!
- Several times from 1927 onward Einstein devises ingenious thought experiments which he thinks can get around the uncertainty relations; he is refuted by Bohr every time.
- In 1932 Einstein nominated Heisenberg for the Nobel; there was nothing petty about Einstein!



Fig. 1:
Einstein and Bohr in Debate, ca. 1925

Other major developments at this time:

- Under the leadership of Bohr and Heisenberg, the Copenhagen Interpretation of quantum mechanics takes definite form, and after the 1927 Solvay Conference it becomes the dominant interpretation of quantum mechanics; other interpretations are “not even wrong.”
- Einstein sarcastically refers to the Copenhagen Interpretation as a “soft pillow that true believers can rest their heads upon.”
- The Copenhagen Interpretation is based on Bohr’s Principle of Complementarity, according to which quantum phenomena have complementary descriptions (wave and particle, position and momentum. . .) both of which are needed, but which are inconsistent if one assumes that both hold in all experimental circumstances.
- Dirac and others extend quantum mechanics to the relativistic treatment of the electron, and lay the foundations for quantum electrodynamics and field theory.
- In 1932 Chadwick discovers the neutron, and most physicists turn their attention to applications of the formalism as it had by then been codified by Dirac and von Neumann; few continue to worry about foundational questions; there were too many juicy applications ready to be picked.
- Schrödinger however, continues to worry about the meaning of the theory; in 1935 he coins the term “entanglement” to describe the mysterious way that

quantum systems can remain *too* correlated even after they have physically separated from each other; Schrödinger (incorrectly!) conjectures that entanglement fades with distance.

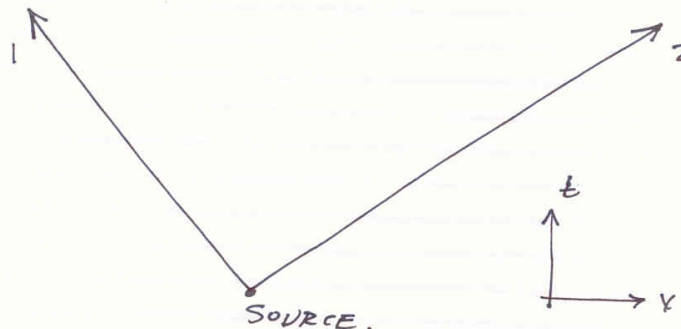
Einstein's last kick at the quantum can (1935):

- In collaboration with B. Podolsky and N. Rosen, Einstein publishes an enigmatic paper called “Can quantum mechanical description of physical reality be considered complete?”
- This is the famous EPR paper, where the letters stand either for the names of the authors, or “Elements of Physical Reality.”
- Probably one of the most cited scientific papers of the 20th century, it is also one of the most difficult.

The EPR Argument

- EPR write down a wave function for an entangled pair of particles whose *difference in position* commutes with their *total momentum*.

- Thus these two global properties can have simultaneous “reality” (definite values).
- We suppose that the particles interact briefly and then separate to arbitrary distances. (See Fig. 1.)

$$\psi(x_1, x_2) = \int_{-\infty}^{\infty} e^{(2\pi i/\hbar)(x_1 - x_2 + x_0)p} dp$$


The diagram illustrates a particle interaction process. A central point is labeled "SOURCE". Two arrows originate from this source: one pointing towards the upper left, labeled with a subscripted "1", and another pointing towards the upper right, labeled with a subscripted "2". To the right of the source, a small coordinate system is shown with a vertical axis labeled $\frac{h}{p}$ and a horizontal axis labeled x .

Fig. 1:
The EPR Apparatus

- Set it up so that Bob and Alice both make their measurements at 12:00 noon in the “lab” frame.
- Bob can’t measure position and momentum of his particle in one operation, but he can choose to measure either.
- Suppose he measures position; then because difference of position is conserved, he can calculate the position of Alice’s particle.
- Suppose, however, Bob chooses to measure the momentum of his particle; then because total momentum is conserved he can calculate the momentum of Alice’s particle.
- Now the crucial assumption is that because the two particles are widely separated, nothing that Bob does at 12:00 noon can influence Alice’s particle *at 12:00 noon*. (Of course, there could be retarded effects.)
- Since Bob could have worked out *either* the position or the momentum of Alice’s particle, Alice’s particle must have *already* possessed definite values of position and momentum even though quantum theory says it cannot.

- EPR conclude by conceding that perhaps that we cannot say that a particle has a definite value of a property until we have measured that property. (This was Bohr's position.)
- However, that would “make the reality of [Alice's particle] depend upon the process of measurement carried out on [Bob's] system, which [by assumption] does not disturb the first system in any way. No reasonable definition of reality could be expected to permit this.” (!!!)
- Einstein's argument thus rested on the *physical assumption* that if we allow the two particles to separate to an appreciable distance, they cannot influence each other at a speed faster than that of light. This is sometimes called “Einstein locality.”

The follow-up:

- Bohr published an immediate reply, which is very difficult to understand; essentially Bohr said that *of course* the two particles don't actually influence each other instantaneously, but at the same time Einstein had asked for more than QM can deliver: the two experimental contexts (measurements of position vs. momentum) cannot be compared.

- Most physicists concluded either that Bohr had settled the debate and it was time to move on, or that the debate was one of those unresolvable philosophical puzzles that one reserves for the pub after a hard day in the lab.

Bohm and Bell

- In 1951 David Bohm publishes a version of the EPR experiment using correlated spin measurements; the virtue of Bohm's version of the experiment is that it could actually be performed.
 - This is basically because spin measurements are discrete.
- Bohm also produces a so-called “hidden variables” version of non-relativistic quantum mechanics in which there are actual superluminal influences (through the quantum potential); Einstein doesn't like it, apparently because of its nonlocality.
- However, Bohm is blacklisted because of his refusal to testify before HUAC and has to move to Brazil.



Fig. 1:
David Bohm



Fig. 1:
John S. Bell

To 1964:

- John S. Bell is puzzled by the fact that Bohm had done what von Neumann and other quantum experts had said was impossible, which was to find an apparently realistic underpinning for quantum statistics.
- However, Bell noted that Bohm's theory was nonlocal, and wondered if there was some way to tell whether *any* conceivable "completion" of quantum mechanics had to be nonlocal.
- Using Bohm's version of EPR, Bell shows that if the two particles do not influence each other in any way after they separate or are emitted from the source, then correlation coefficients between possible measurements that Alice and Bob could make must obey a certain mathematical inequality, now called "Bell's Inequality (BI)."
 - There are many such inequalities, for different types and combinations of entangled particles.
- Bell also shows that quantum mechanics predicts that the BI will be violated; that is, sometimes the particles will be *more* correlated (or anti-correlated) than they could possibly be if locality is correct.
 - The correlation coefficient between spin measurements (that is, the fraction of spin measurements on the particles that agree) is given (depending on the

details of the state) by a sinusoidal function such as $\cos \theta_{AB}$, where θ_{AB} is the relative angle between the detectors.

- Correlations such as this can in principle be easily measured by doing run after run of the EPR apparatus, and compiling statistics of the results.
 - Bell had therefore shown how to do an experimental test of Einstein's locality hypothesis.
- Bell later said, "The theorem seems to show that something is travelling faster than light, although it pains me to say so."
 - In 1982, Alain Aspect and co-workers carried out an EPR experiment which confirmed Bell's prediction to considerable accuracy.
 - Some still doggedly try to find loopholes in Bell's argument, but it is now generally accepted that he showed that quantum mechanics is nonlocal in the precise sense that it violates the BI.
 - Bell did *not* receive a Nobel, or perhaps I should say "No-Bell" Prize.

But what does it mean to violate the BI?

- The most obvious interpretation is that some sort of influence really is moving faster than light between the particles, but not all experts want to put it this way.

- What Einstein wanted to believe was that each particle (which could have a complex internal structure for all we know) carried some sort of coding which would instruct it how to answer all of the possible experimental questions which could be put to it, in such a way as to agree with the formal predictions of quantum mechanics.
- What Bell showed was that the particles can't always give us the right answers unless in some cases they know the answers that the other particles are going to give!
- Sometimes experts will say that Bell's Theorem rules out "common cause" explanations of the correlations:
 - I look rather like my brother not because of instantaneous influences, but because we share a common genetic heritage.
 - Einstein and other believers in locality wanted to think that the correlations between the particles are due to common causes that encoded information in them at the source.
 - The point is that this is mathematically impossible.
- Another way to express Bell's Theorem is to say that in systems of entangled particles emitted from a common source, there can be *more information* encoded in the correlations between the particles than could have been put in at the source.

- Equivalently, the entropy of entangled particles is less than the entropy of particles in a so-called “product” state.
- Some authors put it this way: Bell’s Theorem shows that quantum mechanics is *non-Boolean*. This means that there is no simultaneous consistent valuation of all of the possible experimental questions that can be asked of the particles.
- Bell’s Theorem is a special case of a more general result called the Kochen-Specker theorem, which shows that quantum mechanics cannot have a Boolean underpinning. (See Bub 1997.)
- Around 1990, Itamar Pitowsky showed that the BI were in fact discovered in the 1850s by George Boole, who called them “conditions of possible experience” — i.e., consistency conditions that must hold for observations of properties made on a set of objects with definite, pre-existent properties.
- Bell’s Theorem is often said to rule out “local realism,” the view that the particles must possess a complete set of localizable properties independently of how they are observed. Abner Shimony called Bell’s Theorem “experimental metaphysics.”
- The safest way to interpret Bell’s Theorem is as a purely negative result, in that it rules out local realistic/common causes of quantum correlations, without offering any alternative explanation for them. Some physicists would prefer to say that the particles are correlated simply because “that’s what the calculation says.”

What about a “Bell Telephone”?

- First the accepted view, and then we’ll raise some questions about it.
- Consider an EPR set-up with Bob and Alice measuring pair after pair of entangled particles set out from the source.
- Suppose they agree in advance that Bob will leave his detector at a constant angle; then Alice can vary her detector angle from run to run.

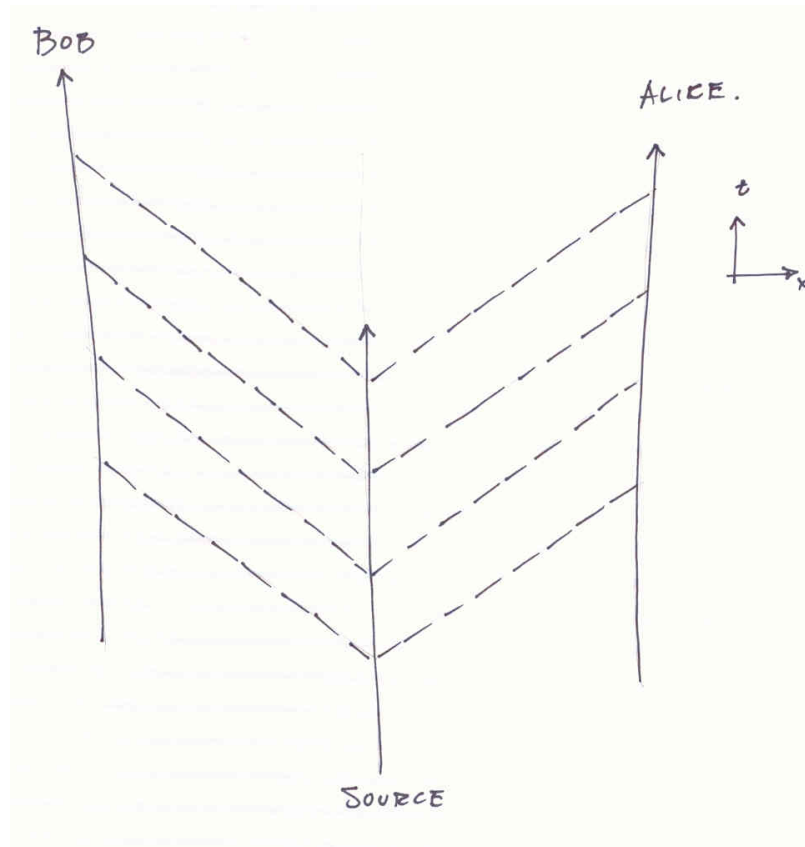


Fig. 2:
A Bell Telephone?

- What each experimenter gets is an apparently random sequence of spins up or down, like a coin toss.
- Nevertheless, when Bob and Alice *later* compare their results, they discover that the BI-violating correlations stand between them.
- It is like a very noisy telephone in which there is so much static that you cannot hear the other person, but where the crackles of static turn out to be correlated.
- Alice wonders if she could send a message to Bob using quantum correlations. She varies her detector angle in step with some code.
- What she can do is vary the *correlation coefficient* in step with her code, but Bob cannot read the message from his local results alone (which continue to look like a random string). That is, Alice can impose a message on the correlations.
- They have to have both sets of results to read the correlations; and so they have to communicate by normal (no faster than light) means in order to read the correlations.
- This is the basis of *quantum cryptography*: you can build a message into quantum correlations, and each set of local results acts as the key for the other; it is considered to be the most theoretically perfect means of message encryption.
- But Alice gets frustrated; she thinks that all she has to do is force her particle to go up, and then (so long as they have the right detector angle) Bob's particle will go down, and she can send a message directly to Bob, faster than light.

- There is no question that Alice can put in some arrangement of magnets, etc., that will force her particle to go any way she wants it to go—but if she does this, she *destroys the nonlocal correlations* and the correlations will then obey a BI.
- Very frustrating!
- Question: must this always be the case; that is, would it never be possible for Alice and Bob to use quantum nonlocality to send a faster-than-light message controllably?

The generally accepted answer is: **NO WAY!**

- The statement that controllable superluminal messaging is impossible is called the No-Signalling Theorem (NST).
- Many proofs have been published in the literature.
- On the basis of these proofs, Abner Shimony said that QM and relativity are in a relationship of “peaceful coexistence”—an ironic phrase suggesting superficial agreement despite an underlying difference in ideology.
- Shimony made a distinction between *controllable* and *uncontrollable* nonlocality; it can be shown that a violation of uncontrollable nonlocality is consistent with

the violation of the BI; the NST supposedly establishes that there is no controllable nonlocality.

- A small minority of authors (including me) think that the NST proofs are no good because they are question-begging; i.e., circular.

Are the NST Proofs Circular?

- There are several mathematical approaches used in these proofs.
- Some appeal to a rule used in local quantum field theory called *microcausality* or *local commutativity*, which states that observations performed at a spacelike separation always commute.
- The problem is that microcausality is a “patch” introduced into field theory precisely in order to forestall predictions of signalling; it is not at all clear that it is generally valid.
- Some NST proofs simply assume from the outset that Bob’s measurements do not affect Alice’s particles; this easily gives a no-signalling result! (B. Hepburn: “An operator that does not act on a wavefunction does not affect the wavefunction.”)
 - Such proofs amount to little more than consistency checks of the formalism.

- Some NST proofs assume that the dynamics of entangled states is fully local; however, Bohm showed that the dynamics of entangled particles is also entangled, and there are other reasons to think that some of the energy of an entangled system is nonlocal.
- Upshot: while we certainly do not yet know how to produce a signalling state, in my (minority!) opinion we do not know that this is generally impossible.

Signalling via Quantum Non-Demolition?

- Y. Aharonov and others have recently (2004) studied spin-spin interactions in systems of two spin-1/2 neutral particles, and apparently shown that limited signalling is possible using *protective measurements*.
- This is a measurement which uses an adiabatic perturbation of part of the system, meaning a perturbation that is applied so slowly that it does not collapse the state.
- The “signalling” they achieve is only operational over very short distances, but it is an interesting counter-example to the usual view.
- The real reason why nonlocality is normally uncontrollable may be because the dynamics of the entangled state is “fragile” (the nonlocal interaction is very weak

normally); it is therefore easily perturbed by standard measurement procedures.

- If this is correct then the key to signalling would be either to create very robust entangled states (perhaps using many particles, as in a BE condensate) or as in Aharonov's scheme using a very gentle measurement interaction.

Signalling and the Quantum Computer

- One of the current “Holy Grails” of physics is to create a practical quantum computer.
- It is not generally realized that the problem of creating a quantum computer is very similar to the problem of signalling; in both cases, one has to extract information from an entangled system without collapsing the system.
- Perhaps one could even say that a quantum computer would be a signalling device, or vice versa!
- Moral of the story: if we want to build quantum computers, we should take the problem of signalling seriously!

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