What Did J. S. Bell Know that George Boole Did Not?

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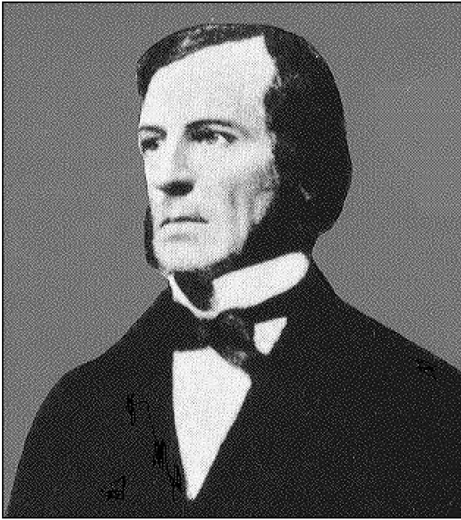
Abstract

One way of expressing the difference between classical and quantum physics is that the latter violates *Booleanity*, which intuitively is a set of constraints on possible measurement outcomes that follow from the assumption that it is possible to observe a system without influencing it. I will explain the notion of Booleanity, outline how Bell showed that it is violated by correlations on entangled particles, and discuss some of its implications for recent work on entanglement and cosmology.

Who Was George Boole?

- George Boole (1815–1864)
  - Enunciated his Laws of Thought; many contributions to modern logic.
  - Boolean logic: each possible proposition has a definite truth value, either T or F (1 or 0); binary "classical" logic.



Who Was John S. Bell?

- John Stewart Bell (1928–1990)
  - His theorem (1964) has been called (H. P. Stapp) "the most profound discovery of science."
  - His theorem is usually stated as follows: QM violates *local realism*.
  - Locality: no faster-than-light influences.
  - Realism: QM statistics can be underpinned by Boolean model.

From EPR to Bell

- Before 1910 Einstein realized that there was something funny about the statistics of light quanta; they were *too well* correlated for their own good!
- Einstein regarded quantum *Spukhaftefernwirkungen* as absurd, literally a threat to scientific rationality.
- EPR paper of 1935: in simple terms, argued that the *apparent* action at a distance of QM was due to the incompleteness of the theory; there had to be a fully local story ("quantum DNA") explaining why particles are correlated the way that QM (successfully) predicts.
- Von Neumann (1930s) argues that "hidden variable" underpinnings of quantum statistics are mathematically impossible.
- Bohm (1952) proves him wrong, but Bohm's theory depends on nonlocal dynamics *via* the quantum potential.

- J. S. Bell (1964) wondered if *any* "completion" of QM had to be nonlocal.
- He studied spin correlations in a version of the EPR thought experiment devised by Bohm, and showed that if such correlations are determined by local hidden variables they have to obey certain inequalities.
- These inequalities are violated by QM correlations; there is no quantum DNA.
- Physicists were in a "schizophrenic" (G. Fleming) state over this until about 1980, when experiments by Aspect *et al.* confirmed Bell's prediction.
- Meanwhile, mathematical work by Gleason (1957), Kochen and Specker (1967), showed that predictions of QM are in general inconsistent with Boolean underpinning of quantum statistics.
- Bell's Theorem is special case of generalized Bell-KS Theorem for spacelike separate entangled particles (Bub, Mermin).

Quick Look at the Math

- Bell's Theorem applies to entangled states (aka Bell states, ebits), those which cannot be written as tensor products of kets of individual particles.
  - Important example is the singlet state (of spin-$\frac{1}{2}$ fermion such as electron)

$$| \Psi_S \rangle = \frac{1}{\sqrt{2}}\Big(| +- \rangle - | -+ \rangle\Big).$$

  - Existence of entanglement is demanded by superposition principle: (in general) all linear combinations of kets are allowed kets.
  - Correlations between measurements on entangled particles in general have sinusoidal forms; for singlet it is just $-\cos\theta_{AB}$, where θ_{AB} is the *relative* angle between Alice and Bob's detectors.

Quick Look at the Math (con't)

- Let $P(a, b)$ be the correlation coefficient between Alice and Bob's measurement results on successive pairs of entangled particles, where a and b are their detector parameters.
  - Define $\Delta_{AB} \equiv |P(a, b) - P(a, b')| + |P(a', b) + P(a', b')|$.
  - One can show that given local realism,

    $$\Delta_{AB} \leq 2.$$

    (This is Clauser-Horn-Shimony-Holt version of BI.)
  - However, for the singlet state, $\Delta_{AB} \leq 2\sqrt{2}$. Therefore, local realism must in some way be false.

Logical Structure of No-Go Theorems

- We arrive at a paradox if we assume that a quantum system has a Boolean property structure.
- Bell's (1964) argument was a *modus tollens*: P implies Q, not-Q, therefore not-P.
- "For Whom Bell's Theorem Tolls" is therefore a double pun.
- No-go theorems are negative results; they show what cannot be the case, but they do not directly show us what must be the case.

What is Booleanity?

- This can be expressed informally in several equivalent ways:
  - To say that something has a Boolean property structure is to say that its properties obey set theory.
  - Every possible proposition about the properties of that thing has a definite truth value (even if one might not know them for practical reasons).
  - There is a logically consistent description of the whole of the thing.
  - Every possible experimental question we could ask about the thing already has an answer *before we ask it*.
  - The properties of the thing can be described by "urn" model (Pitowsky 1994).
    - Pitowsky showed that the Bell Inequalities were first discovered by George Boole in the 1850s, and were called by Boole "conditions of possible experience."
    - The Boole-Bell Inequalities are simply consistency conditions *given that* one examines a system *without altering its properties*.

So What is Non-Booleanity?

- Bell-KS Theorem: QM correlations violate set theory (since Bell-Boole Inequalities follow from set theory).
- Demopoulos (2004) emphasizes that descriptions of quantum systems are *incompletable* because the presumption of completability entails a mathematical contradiction.
- This was anticipated by Schrödinger (1935):

> *. . . if I wish to ascribe to the model [of a quantum mechanical oscillator] at each moment a definite (merely not known exactly to me) state, or (which is the same) to* **all** *determining parts definite (merely not known exactly to me) numerical values, then there is no supposition as to these numerical values* **to be imagined** *that would not conflict with some portion of quantum theoretical assertions.*

The Lump In the Carpet

- Fitting QM predictions to a Boolean substrate is like trying to smooth out a carpet cut from surface of sphere onto a flat floor.
  - There will be a lump!
  - You can move it around and even hide it under furniture, but you can't make it go away.
- It is not entirely accurate to call the "no-go" theorems "no hidden variable" theorems; more accurately, they are *no Boolean variable* theories.
  - Even more precisely, *not enough Boolean variable theorems*, since non-Boolean quantum systems can have Boolean subspaces defined by CSCO.

Directions for Research

- Why Non-Booleanity?
- Non-Booleanity in Dynamics of Entangled States
- Implications for Spacetime Structure

Why Non-Booleanity?

- Non-Booleanity is formally a consequence of the fact that quantum mechanical observables come in non-commuting conjugate pairs (Bub):
  - $[\hat{A}, \hat{B}] = i\hbar\hat{C}$.
- But a clearer explanation of this connection is needed.
- Leads to deeper question about non-commutativity: "who ordered *that*?"
  - Why do some observables not commute?
  - Why does Planck's constant of action have the observed value it has (6.626×10^{-27}erg.sec)?

Phase Quantization and Quantum Gravity

- Crazy idea: finite rotations do not commute; possibly non-commutativity is reflection of quantization of angle/phase.
- The notion that all rotations are finite is consistent with DSR and quantum gravitational theories in which spacetime is discrete.
- But which comes first?
  - Are rotations finite because spacetime is discrete?
  - Or is spacetime discrete because rotations (when better understood) are finite?
- Stay tuned!

Non-Booleanity in Dynamics of Entangled States

- Doctrine of "peaceful coexistence" between QM and relativity (Shimony) depends upon assumption (never proven) that dynamics of particles are local (and thus Boolean) even if the particles are entangled.
- Is this plausible?
  - Recent work by M. Hotta (quantum energy teleportation) and M. Sarovar *et al.* (on entanglement in photosynthetic biomolecules) involves entangled Hamiltonians.
  - It is highly likely that such energy states are non-Boolean, but this requires further clarification.

Implications for Spacetime Structure

- Conventional QM and field theory treats quantum behaviour in a classical C^∞ Minkowski background; such structures are Boolean.
- The project of quantum gravity is to treat spacetime itself as a *quantum object*.
  - Is there a Bell's Theorem for the universe as a whole, showing that it cannot have a Boolean property structure?
  - This would amount to a no-go theorem for the block universe, and would thereby show that static interpretations of time are untenable.
  - Such a theorem might be accomplished by using quantum interferometry.

Example: Wheeler/Supina Delayed Choice Experiment

- Wheeler proposed an interferometer extending several billion ly, with the light from a distant quasar bent around a gravitational lens.
- A measurement choice in an Earthly lab seems to determine which path some photons took even though they were emitted billions of years ago.
- This can be set up as a Bell-KS experiment, which will demonstrate correlations that will violate the assumption that the photons had definite trajectories.
- Well-verified set-ups like this show that the *past* (or indeed any region of spacetime) is to some degree non-Boolean!
- A work in progress!