

Answers to Exercise 1.3 Logarithms

1.

(a) $x = \log(10) = \log(10^1) = 1$

(b) $x = \log(0.01) = \log(10^{-2}) = -2$

(c) $x = \log\left(\frac{1}{1000}\right) = \log(10^{-3}) = -3$

2.

(a) $\log(10^{-pH}) = -pH$

(b) $\log(10x^2) = \log(10) + \log(x^2) = \log(10) + 2\log(x) = 1 + 2\log(x)$

(c) $e^{\ln(x)} = x$

3.

Each answer is reported first with more digits than are significant so that you can check that you got the same answer in your calculator. Then it's rounded to the correct number of sig. fig. as described in your lab manual.

(a) $13.2 = -\log(x)$
 $-13.2 = \log(x)$
 $10^{-13.2} = 10^{\log(x)}$
 $10^{-13.2} = x$
 $x = 6.31 \times 10^{-14}$
 $x = 6 \times 10^{-14}$

*Note that 6E-14 is **not** an acceptable way to write this.*

(b) $13.2 = -\ln(x)$
 $-13.2 = \ln(x)$
 $e^{-13.2} = e^{\ln(x)}$
 $e^{-13.2} = x$
 $x = 1.85 \times 10^{-6}$
 $x = 2 \times 10^{-6}$

Comparing the answers to (a) and (b), you can see why using the correct base for your logarithm matters!

$$\begin{aligned}
 \text{(c)} \quad \ln\left(\frac{x}{5}\right) &= 1.50 \\
 \frac{x}{5} &= e^{1.50} \\
 x &= 5 \cdot e^{1.50} \\
 x &= 22.408 \\
 x &= 22
 \end{aligned}$$

$$\begin{aligned}
 \text{(d)} \quad \ln\left(\frac{x}{5}\right) &= -1.50 \\
 \frac{x}{5} &= e^{-1.50} \\
 x &= 5 \cdot e^{-1.50} \\
 x &= 1.11565 \\
 x &= 1.1
 \end{aligned}$$

Comparing the answers to (c) and (d), you can see why the negative sign matters!

In (c), x is greater than 5. In (d), x is less than 5.

Logarithms of fractions (less than 1) give negative numbers while logarithms of numbers greater than 1 give positive numbers. (Logarithms of negative numbers give error messages from your calculator...)

$$\begin{aligned}
 \text{(e)} \quad 13.2 &= e^x \\
 \ln(13.2) &= x \\
 x &= 2.580
 \end{aligned}$$

$$\begin{aligned}
 \text{(f)} \quad 13.2 &= e^{x^2} \\
 \ln(13.2) &= x^2 \\
 \sqrt{\ln(13.2)} &= x \\
 x &= 1.606
 \end{aligned}$$