Answers to Exercise 2.4
Energy of Nuclear Reactions

1. **Step 1: Write a balanced equation for the reaction**

$$\mathbf{2\,{}^{12}_{6}C \rightarrow {}^{20}_{10}Ne + {}^{4}_{2}He}$$

Step 2: Calculate the difference between masses of products and reactants

Recall that if the mass of an atom is 12 u then the molar mass of that element is 12 g/mol.

$$\Delta m = \sum m_{products} - \sum m_{reactants}$$

$$\Delta m = \left(m_{{}^{20}_{10}Ne} + m_{{}^{4}_{2}He}\right) - \left(2 \times m_{{}^{12}_{6}C}\right)$$

$$\Delta m = \left(19.992\,440\,176\,\frac{g}{mol} + 4.002\,603\,254\,\frac{g}{mol}\right) - \left(2 \times 12.000\,000\,000\,\frac{g}{mol}\right)$$

$$\Delta m = -0.004\,956\,570\,\frac{g}{mol}$$

Step 3: Calculate the energy change for the reaction in J/mol *answer to part (a)*

Remember to convert your mass change into kg (or into kg/mol) since that is the only way to convert the final unit into J (or into J/mol). 1 J = 1 kg·m^2/s^2

$$\Delta E = \Delta m c^2$$

$$\Delta E = \left(-0.004\,956\,570\,\frac{g}{mol}\right)\left(2.997\,925 \times 10^8\,\frac{m}{s}\right)^2\left(\frac{1kg}{1000g}\right)\left(\frac{1J}{1\frac{kg\cdot m^2}{s^2}}\right)$$

$$\Delta E = -4.454\,744 \times 10^{11}\,\frac{J}{mol}$$

Step 4: Calculate the energy change for the reaction in J *answer to part (b)*

Recall that there are 6.022141×10^{23} of anything in 1 mol.

$$\Delta E = -4.454\,744 \times 10^{11}\,\frac{J}{mol} \times \frac{1\,mol}{6.022\,141 \times 10^{23}} = -7.397\,276 \times 10^{-13} J$$

Step 5: Check your work

Does your answer seem reasonable? Are sig. fig. correct?

ΔE is negative, indicating that energy is released. We expect energy to be released in most nuclear reactions.

*The value for ΔE in J/mol is __**much**__ larger than the value for ΔE in J. This is to be expected since there are __**many**__ atoms in one mole. If many atoms react, that should release much more energy than if just one atom reacts.*

For comparison, chemical reactions that are not nuclear reactions tend to have energy changes which range from 0 kJ/mol to a few thousand kJ/mol (values in the hundred kJ/mol range are common). So, this nuclear reaction releases about a million times more energy than many conventional chemical reactions. (10^{11} J/mol ÷ 10^5 J/mol = 10^6)

The nuclear reactions in questions 2 and 3 of this exercise have energy changes with similar orders of magnitude (10^{11} or 10^{12} J/mol).

2.

(a) $^{26}_{13}Al + ^{0}_{-1}e \rightarrow ^{26}_{12}Mg$

(b) reactants: ^{26}Al has 13 electrons. The electron "captured" by the Al nucleus is from the Al atom, so it is not counted separately.

products: ^{26}Mg has 12 electrons.

The electron "captured" by the Al nucleus reacted with a proton in the nucleus to make a neutron, so its mass has already been accounted for within the masses of ^{26}Al and ^{26}Mg. We therefore do not include its mass (again) in the calculation of Δm for this reaction.

When considering the "big picture", <u>do not</u> include the masses of electrons/beta particles or positrons when calculating Δm in order to calculate ΔE for nuclear reactions.

<u>Do</u> include the masses of all atoms as well as alpha particles.

(c) **Step 1: Write a balanced equation for the reaction**

see part (a)

Step 2: Calculate the difference between masses of products and reactants

$$\Delta m = \sum m_{products} - \sum m_{reactants}$$

$$\Delta m = m_{^{26}_{12}Mg} - m_{^{26}_{13}Al}$$

$$\Delta m = \left(25.982\ 592\ 968\frac{g}{mol}\right) - \left(25.986\ 891\ 904\frac{g}{mol}\right)$$

$$\Delta m = -0.004\ 298\ 936\frac{g}{mol}$$

Step 3: Calculate the energy change for the reaction in J/mol

$$\Delta E = \Delta mc^2$$

$$\Delta E = \left(-0.004\ 298\ 936\frac{g}{mol}\right)\left(2.997\ 925 \times 10^8\frac{m}{s}\right)^2\left(\frac{1kg}{1000g}\right)\left(\frac{1J}{1\frac{kg\cdot m^2}{s^2}}\right)$$

$$\Delta E = -3.863\ 692\ \times 10^{11}\frac{J}{mol}$$

Step 4: Calculate the energy change for the reaction in J

$$\Delta E = -3.863\ 692 \times 10^{11}\frac{J}{mol} \times \frac{1\ mol}{6.022141 \times 10^{23}} = -6.415\ 811 \times 10^{-13}J$$

Step 5: Check your work

Does your answer seem reasonable? Are sig. fig. correct?

ΔE is negative, indicating that energy is released. We expect energy to be released in most nuclear reactions.

*The value for ΔE in J/mol is **<u>much</u>** larger than the value for ΔE in J. This is to be expected since there are **<u>many</u>** atoms in one mole. If many atoms react, that should release much more energy than if just one atom reacts.*

3.

(a) $4\,{}^{1}_{1}H \rightarrow {}^{4}_{2}He + 2\,{}^{0}_{+1}\beta$

(b) reactants: Each ^{1}H has one electron, so there are four electrons in total.

products: ^{4}He has two electrons, so there are two electrons in total.

Therefore, two electrons have "gone missing" between the reactants and products.

Those two electrons are annihilated by the two positrons.

A positron is the antimatter equivalent of an electron. When a particle of matter and its antimatter equivalent come into contact, their entire mass is converted into energy.

Because the two positrons will be annihilated as soon as they come into contact with the two "missing" electrons, we do not include their mass in the calculation of Δm for this reaction.

When considering the "big picture", <u>do not</u> include the masses of electrons/beta particles or positrons when calculating Δm in order to calculate ΔE for nuclear reactions.

<u>Do</u> include the masses of all atoms as well as alpha particles.

(c) **Step 1: Write a balanced equation for the reaction**

see part (a)

Step 2: Calculate the difference between masses of products and reactants

$\Delta m = \sum m_{products} - \sum m_{reactants}$

$\Delta m = m_{{}^{4}_{2}He} - \left(4 \times m_{{}^{1}_{1}H}\right)$

$\Delta m = \left(4.002\ 603\ 254\,\frac{g}{mol}\right) - \left(4 \times 1.007\ 825\ 032\,\frac{g}{mol}\right)$

$\Delta m = -0.028\ 696\ 874\,\frac{g}{mol}$

Step 3: Calculate the energy change for the reaction in J/mol

$\Delta E = \Delta mc^2$

$\Delta E = \left(-0.028\ 696\ 874\,\frac{g}{mol}\right)\left(2.997\ 925 \times 10^{8}\,\frac{m}{s}\right)^{2}\left(\frac{1kg}{1000g}\right)\left(\frac{1J}{1\frac{kg\cdot m^2}{s^2}}\right)$

$\Delta E = -2.579\ 147\ 1 \times 10^{12}\,\frac{J}{mol}$

Step 4: Calculate the energy change for the reaction in J

$\Delta E = -2.579\ 147\ 1 \times 10^{12}\,\frac{J}{mol} \times \frac{1\ mol}{6.022141 \times 10^{23}} = -4.282\ 774 \times 10^{-12} J$

Step 5: Answer the question

The energy released by this reaction is $4.282\ 774 \times 10^{-12} J$ or $2.579\ 147\ 1 \times 10^{12}\,\frac{J}{mol}$.

Step 6: Check your work

Does your answer seem reasonable? Are sig. fig. correct?

ΔE is negative, indicating that energy is released. We expect energy to be released in most nuclear reactions.

*The value for ΔE in J/mol is <u>**much**</u> larger than the value for ΔE in J. This is to be expected since there are <u>**many**</u> atoms in one mole. If many atoms react, that should release much more energy than if just one atom reacts.*