Answers to Exercise 2.7
Kinetics of Nuclear Reactions

1.

(a) $^{210}_{84}Po \rightarrow\, ^{206}_{82}Pb + \,^{4}_{2}\alpha$

(b) **Step 1: Calculate the half-life of ^{210}Po**

$ln(2) = k \cdot t_{1/2}$

$t_{1/2} = \frac{ln(2)}{k} = \frac{ln(2)}{0.00501\ d^{-1}} = 138\ d$

Step 2: Check your work

Does your answer seem reasonable? Are sig. fig. correct?

Half-lives should have units of time. Days are therefore an appropriate unit for half-life.

Recall that A = kN, so k represents the probability of each nuclide decaying within a given time. A = observed activity (# decays per unit time); N = # nuclides; k = decay constant

So, since k ≈ 0.005 d^{-1}, we can expect that about 5 of every 1000 nuclides decay per day. This is the same as saying that 1 of every 200 nuclides should decay per day.

*It is therefore reasonable for it to take at least 100 days for half the nuclides to decay. Remember that this is *not* a linear relationship! As some nuclides decay, there are fewer left to react – but it can give a general sense of whether or not an answer is reasonable.*

(c) **Step 1: Calculate the number of moles of ^{210}Po in the sample**

$n_{Po-210} = 1.00\ g\ \times \frac{1\ mol}{209.982874\ g} = 0.00476\ mol$

Step 2: Calculate the number of atoms of ^{210}Po in the sample

$N_{Po-210} = 0.00476\ mol\ \times \frac{6.022141\times 10^{23}\ atoms}{1\ mol} = 2.87 \times 10^{21}\ atoms$

Step 3: Calculate the activity of the sample

$A = kN = (0.00501 d^{-1})(2.87 \times 10^{21}) = 1.44 \times 10^{19} d^{-1}$

This is one of the very few times when you will see units "disappear" in a calculation in CHEM 1000. N is the number of atoms. A is the number of decays observed. To make the units work, you would need to introduce an additional conversion factor stating that each decay comes from one atom. That would give you an answer with units of decays per day:

$A = kN = (0.00501 day^{-1})(2.87 \times 10^{21} atoms)\left(\frac{1\ decay}{1\ atom}\right) = 1.44 \times 10^{19}\, \frac{decays}{day}$

The convention, however, is to simply report activities in units of inverse time with the "decays per…" part taken as understood.

Step 4: Check your work

Does your answer seem reasonable? Are sig. fig. correct?

*There are a *lot* of atoms in a 1.00 g sample. If 0.5% of them decay each day, we expect the activity to be a relatively large number of decays per day.*

(d) **Step 1: Organize your information**

$k = 0.00501\ d^{-1}$

$N_1 = 2.87 \times 10^{21}\ atoms$ $\quad N_2 = ???$

$A_1 = 1.44 \times 10^{19} d^{-1}$ $\quad A_2 = ???$

$t_1 = 0\ d$ *(no time passed)* $\quad t_2 = 365\ d$ *Make sure units for time and k "match"*

Step 2: Calculate the number of atoms of ^{210}Po left after 365 days (N_2)

$ln\left(\frac{N_2}{N_1}\right) = -k(t_2 - t_1)$

$ln\left(\frac{N_2}{N_1}\right) = -(0.00501 d^{-1})(365\ d - 0\ d)$

$ln\left(\frac{N_2}{N_1}\right) = -1.83$

$\frac{N_2}{N_1} = e^{-1.83}$

$N_2 = N_1 \cdot e^{-1.83} = (2.87 \times 10^{21}\ atoms)(e^{-1.83}) = 4.61 \times 10^{20}\ atoms$

Step 3: Calculate the activity of the sample (A_2)

$A_2 = kN_2 = (0.00501 d^{-1})(4.61 \times 10^{20}) = 2.31 \times 10^{18} d^{-1}$

Step 4: Check your work

Does your answer seem reasonable? Are sig. fig. correct?

As expected, both the number of atoms and activity of ^{210}Po are lower after time has passed.

Alternative approach:

Step 1: Organize your information

see first approach

Step 2: Combine the two equations used in Steps 2 & 3 above to generate one equation

$A = kN$ *therefore* $N = \frac{A}{k}$ *therefore* $N_1 = \frac{A_1}{k}$ *and* $N_2 = \frac{A_2}{k}$

$ln\left(\frac{N_2}{N_1}\right) = -k(t_2 - t_1)$ *therefore* $ln\left(\frac{\left(\frac{A_2}{k}\right)}{\left(\frac{N_1}{k}\right)}\right) = -k(t_2 - t_1)$

therefore $ln\left(\frac{A_2}{A_1}\right) = -k(t_2 - t_1)$

Step 3: Calculate the activity of ^{210}Po after the sample has decayed for 365 days (A_2)

$ln\left(\frac{A_2}{A_1}\right) = -k(t_2 - t_1)$

$ln\left(\frac{A_2}{A_1}\right) = -(0.00501 d^{-1})(365\ d - 0\ d)$

$ln\left(\frac{A_2}{A_1}\right) = -1.83$

$\frac{A_2}{A_1} = e^{-1.83}$

$A_2 = A_1 \cdot e^{-1.83} = (1.44 \times 10^{19}\ d^{-1})(e^{-1.83}) = 2.31 \times 10^{18}\ d^{-1}$

Step 4: Check your work

Does your answer seem reasonable? Are sig. fig. correct?

As expected, the activity of ^{210}Po is lower after time has passed.

2.

(a) $^{99}_{42}M \rightarrow {}^{99m}_{43}Tc + {}^{0}_{-1}\beta$

(b) $^{99m}_{43}Tc \rightarrow {}^{99}_{43}Tc$ *or* $^{99m}_{43}Tc \rightarrow {}^{99}_{43}Tc + {}^{0}_{0}\gamma$

(c) i. **Calculate the decay constant for ^{99}Mo**

$$ln(2) = k \cdot t_{1/2}$$

$$k = \frac{ln(2)}{t_{1/2}} = \frac{ln(2)}{2.75\ d} = 0.252\ d^{-1}$$

Calculate the decay constant for ^{99m}Tc

$$ln(2) = k \cdot t_{1/2}$$

$$k = \frac{ln(2)}{t_{1/2}} = \frac{ln(2)}{6.01\ h} = 0.115\ h^{-1}$$

Calculate the decay constant for ^{99}Tc

$$ln(2) = k \cdot t_{1/2}$$

$$k = \frac{ln(2)}{t_{1/2}} = \frac{ln(2)}{2.11 \times 10^{5}\ y} = 3.29 \times 10^{-6}\ y^{-1}$$

ii. ^{99}Tc has the longest half-life (and the smallest decay constant) so it is the most stable of the three nuclides.

iii. There are many advantages of using a nuclide with a relatively short half-life in nuclear medicine. They include:

- Nuclides with shorter half-lives have higher decay constants, so they emit more radiation per atom than nuclides with longer half-lives. This means that you can get the amount of radiation needed for the test using a smaller amount of radioactive material.
- Because a higher fraction of the radioactive nuclides decay within a given unit of time, it takes less time for the majority of nuclides to have decayed, so the patient is exposed to radiation for a shorter overall period of time.

Keep in mind that there is a point at which a shorter half-life stops being better. If an isotope has a half-life that is too short, it may not reach the desired part of the body before decaying so much that it no longer emits detectable radiation.

(d) *Recall that 1 Bq = 1 decay per second = 1 s^{-1} It is a unit for measuring activity (A).*

Step 1: Organize your information

$k = 0.115\ h^{-1}$

$A_1 = 1000\ MBq$ $A_2 = 1\ MBq$

$t_1 = 0\ h$ $t_2 = ???$ *Since k is in h^{-1}, it is easiest to measure time in h*

Step 2: Calculate time at which activity has dropped to 1 MBq

$ln\left(\frac{A_2}{A_1}\right) = -k(t_2 - t_1)$ *See answer to 1(d) for derivation*

$$ln\left(\frac{1\ MBq}{1000\ MBq}\right) = -(0.115\ h^{-1})(t_2 - 0\ h)$$

$$ln(0.001) = -(0.115\ h^{-1})(t_2)$$

$$t_2 = \frac{ln(0.001)}{-0.115\ h^{-1}} = \frac{-6.9}{-0.115\ h^{-1}} = 60\ h$$

Step 3: Check your work

Does your answer seem reasonable? Are sig. fig. correct?
The half-life of ^{99m}Tc *is about 6 hours. 60 hours is therefore about 10 half-lives.*
So, you'd expect $(1/2)^{10} \approx 1/1000$ *of the* ^{99m}Tc *to be left after 60 hours.*