Answers to Exercise 3.3
Calculations Based on Bohr's Model of the Atom

1.

(a) The energy of the photon released when Li^{2+} relaxes from the n = 5 to the n = 4 state is equal to the energy difference between an electron with n = 5 and an electron with n = 4. Li^{2+} has three protons so Z = 3.

Step 1: Calculate the energy for Li^{2+} with n = 4

$$E_{n=1} = -R_H \frac{Z^2}{n^2} = -R_H \frac{(3)^2}{(4)^2} = -\frac{9}{16} R_H = -1.226178 \times 10^{-18} J$$

Step 2: Calculate the energy for Li^{2+} with n = 5

$$E_{n=2} = -R_H \frac{Z^2}{n^2} = -R_H \frac{(3)^2}{(5)^2} = -\frac{9}{25} R_H = -7.847539 \times 10^{-19} J$$

Step 3: Calculate the difference in energy between the n = 4 and n = 5 states ($\Delta E = E_{final} - E_{initial}$)

$$\Delta E = E_{n=4} - E_{n=5} = (-1.226178 \times 10^{-18} J) - (-7.847539 \times 10^{-19} J) = -4.41424 \times 10^{-19} J$$

Step 4: Calculate the energy of the photon released

Since the sign of ΔE is negative, energy is released.

The energy of the photon released is 4.41424×10^{-19}J.

Step 5: Check your work

Does your answer seem reasonable? Are sig. fig. correct?

The energy of the photon has a similar order of magnitude to R_H.

(b) **Step 1: Calculate the frequency of the photon released**

$$E = h\nu$$

$$\nu = \frac{E}{h} = \frac{4.41424 \times 10^{-19} J}{6.626070 \times 10^{-34} \frac{J}{Hz}} = 6.66193 \times 10^{14} Hz$$

Step 2: Calculate the wavelength of the photon released

$$c = \nu\lambda$$

$$\lambda = \frac{c}{\nu} = \frac{2.997925 \times 10^{8} \frac{m}{s}}{6.66193 \times 10^{14} Hz} \times \frac{1Hz}{1\frac{1}{s}} = 4.50009 \times 10^{-7} m$$

$$\lambda = 4.50009 \times 10^{-7} m \times \frac{10^9 nm}{1m} = 450.009 nm$$

The two steps may be combined using E=hc/λ

Step 3: Check your work

Does your answer seem reasonable? Are sig. fig. correct?

We expect wavelengths in the UV, visible or IR range for Bohr line spectra calculations.

(c) This is visible light.

2.

(a) The energy of the photon released when He^+ relaxes from the $n = 3$ to the $n = 1$ state is equal to the energy difference between an electron with $n = 3$ and an electron with $n = 1$.

He^+ has two protons so $Z = 2$.

Step 1: Calculate the energy for He^+ with $n = 1$

$$E_{n=1} = -R_H \frac{Z^2}{n^2} = -R_H \frac{(2)^2}{(1)^2} = -4R_H = -8.719488 \times 10^{-18} J$$

Step 2: Calculate the energy for He^+ with $n = 3$

$$E_{n=3} = -R_H \frac{Z^2}{n^2} = -R_H \frac{(2)^2}{(3)^2} = -\frac{4}{9} R_H = -9.688320 \times 10^{-19} J$$

Step 3: Calculate the difference in energy between the $n = 1$ and $n = 3$ states ($\Delta E = E_{final} - E_{initial}$)

$$\Delta E = E_{n=1} - E_{n=3} = (-8.719488 \times 10^{-18} J) - (-9.688320 \times 10^{-19} J) = -7.750656 \times 10^{-18} J$$

Step 4: Calculate the energy of the photon released

Since the sign of ΔE is negative, energy is released.

The energy of the photon released is 7.750656×10^{-18}J.

Step 5: Calculate the frequency of the photon released

$$E = h\nu$$

$$\nu = \frac{E}{h} = \frac{7.750656 \times 10^{-18} J}{6.626070 \times 10^{-34} \frac{J}{Hz}} = 1.169721 \times 10^{15} Hz$$

Step 6: Calculate the wavelength of the photon released

$$c = \nu\lambda$$

$$\lambda = \frac{c}{\nu} = \frac{2.997925 \times 10^{8} \frac{m}{s}}{1.169721 \times 10^{15} Hz} \times \frac{1Hz}{1\frac{1}{s}} = 2.562939 \times 10^{-8} m$$

$$\lambda = 2.562939 \times 10^{-8} m \times \frac{10^9 nm}{1m} = 25.62939 nm$$

Steps 5 and 6 may be combined using $E=hc/\lambda$

Step 7: Check your work

Does your answer seem reasonable? Are sig. fig. correct?

We expect wavelengths in the UV, visible or IR range for Bohr line spectra calculations.

(b) longer

The energy difference between $n = 5$ and $n = 3$ is smaller than the one between $n = 3$ to $n = 1$. Therefore, a lower energy photon will be emitted for the $n = 5$ to $n = 3$ transition. A lower energy photon has a longer wavelength.

3.

(a) He^+

Both are one-electron atoms/ions, but He^+ has two protons in the nucleus so the electron should be more strongly attracted to the nucleus therefore more difficult to remove.

(b) The ionization energy for a ground state hydrogen atom is equal to the energy difference between an electron with $n = 1$ and an electron with $n = \infty$ (the ionization limit). The energy for an electron with $n = \infty$ is 0 J.

A hydrogen atom has one proton so $Z = 1$.

Step 1: Calculate the energy for a hydrogen atom with *n* = 1

$$E_{n=1} = -R_H \frac{Z^2}{n^2} = -R_H \frac{(1)^2}{(1)^2} = -R_H = -2.179872 \times 10^{-18} J$$

Step 2: Calculate the ionization energy for a ground state hydrogen atom.

$$E_i = E_{n=\infty} - E_{n=1} = (0J) - (-2.179872 \times 10^{-18} J) = 2.179872 \times 10^{-18} J$$

Step 3: Check your work

Does your answer seem reasonable? Are sig. fig. correct?

The ionization energy of ground state hydrogen is R_H.

(c) The ionization energy for ground state He^+ is equal to the energy difference between an electron with $n = 1$ and an electron with $n = \infty$ (the ionization limit). The energy for an electron with $n = \infty$ is 0 J.

He^+ has two protons so $Z = 2$.

Step 1: Calculate the energy for He^+ with *n* = 1

$$E_{n=1} = -R_H \frac{Z^2}{n^2} = -R_H \frac{(2)^2}{(1)^2} = -4R_H = -8.719488 \times 10^{-18} J$$

Step 2: Calculate the ionization energy for ground state He^+

$$E_i = E_{n=\infty} - E_{n=1} = (0J) - (-8.719488 \times 10^{-18} J) = 8.719488 \times 10^{-18} J$$

Step 3: Check your work

Does your answer seem reasonable? Are sig. fig. correct?

The ionization energy has a similar order of magnitude to R_H.

(d) Yes.

It took four times as much energy to excite an electron from He^+ than from H.

(e) A neutral He atom has two electrons. The equations used in parts (b) and (c) only work for one-electron atoms/ions. This is because they only account for the attraction between two bodies (the electron and the nucleus). Adding any more electrons introduces more forces that must be accounted for (electron-electron repulsion).

Note that adding protons and/or neutrons does not prevent us from doing these calculations because they just change the charge and mass of the nucleus; they do not introduce new bodies (and therefore new interactions).

4.

(a) **Step 1: Calculate the frequency of the light**

$c = \nu\lambda$

$$\nu = \frac{c}{\lambda} = \frac{2.997925\times10^{8}\frac{m}{s}}{250nm} \times \frac{10^{9}nm}{1m} = 1.20 \times 10^{15}\frac{1}{s} = 1.20 \times 10^{15}Hz$$

Step 2: Calculate the energy of a single photon

$$E = h\nu = \left(6.626070 \times 10^{-34}\frac{J}{Hz}\right)(1.20 \times 10^{15}Hz) = 7.95 \times 10^{-19}J$$

The two steps may be combined using E=hc/λ

Step 3: Check your work

Does your answer seem reasonable? Are sig. fig. correct?

(b) *This calculation was already done in question 3(c) of this exercise!*

The ionization energy for ground state He^+ is equal to the energy difference between an electron with *n* = 1 and an electron with *n* = ∞ (the ionization limit). The energy for an electron with *n* = ∞ is 0 J.

He^+ has two protons so *Z* = 2.

Step 1: Calculate the energy for He^+ with *n* = 1

$$E_{n=1} = -R_H\frac{Z^2}{n^2} = -R_H\frac{(2)^2}{(1)^2} = -4R_H = -8.719488\times10^{-18}J$$

Step 2: Calculate the ionization energy for ground state He^+

$$E_i = E_{n=\infty} - E_{n=1} = (0J) - (-8.719488\times10^{-18}J) = 8.719488\times10^{-18}J$$

Step 3: Check your work

Does your answer seem reasonable? Are sig. fig. correct?

The ionization energy has a similar order of magnitude to R_H.

This is a higher energy than the energy of one photon from the UV lamp.

As such, the UV lamp is NOT capable of exciting the last electron out of a ground state helium atom.