Answers to Exercise 9.5
Calculations Relating Root-Mean-Square Speed and Temperature of Gases

1.

(a) The average kinetic energy for any gas at the same temperature is the same, so it's only necessary to do one calculation for both gases.

Step 1: Convert temperature into Kelvin

$T = 21 + 273.15 = 294.15\ K = 294\ K$

This value only has 3 sig. fig. because 21 ℃ has no decimal places; however, we will still use 294.15 K in the calculation to minimize rounding error.

Step 2: Calculate average kinetic energy

$\overline{E_k} = \frac{3}{2}RT$

$\overline{E_k} = \frac{3}{2}\left(8.314\,462\,\frac{J}{mol \cdot K}\right)(294.15\ K)$

$\overline{E_k} = 3.67 \times 10^3\,\frac{J}{mol} = 3.67\,\frac{kJ}{mol}$

Step 3: Check your work

Does your answer seem reasonable? Are sig. fig. correct?

(b) **Step 1: Calculate molar mass for each gas in kg/mol**

$M_{Ar} = 39.948\ \frac{g}{mol} \times \frac{1\ kg}{1000\ g} = 0.039948\ \frac{kg}{mol}$

$M_{N_2} = 2\left(14.0067\frac{g}{mol}\right) = 28.0134\ \frac{g}{mol} \times \frac{1\ kg}{1000\ g} = 0.0280134\ \frac{kg}{mol}$

Step 2: Calculate root-mean-square speed for each gas

$v_{rms} = \sqrt{\frac{3RT}{M}}$

$v_{rms}(Ar) = \sqrt{\frac{3\left(8.314462\frac{J}{mol \cdot K}\right)(294.15K)}{\left(0.039948\frac{kg}{mol}\right)} \times \frac{1\frac{kg \cdot m^2}{s^2}}{1J}} = 429\,\frac{m}{s}$

$v_{rms}(N_2) = \sqrt{\frac{3\left(8.314462\frac{J}{mol \cdot K}\right)(294.15K)}{\left(0.0280134\frac{kg}{mol}\right)} \times \frac{1\frac{kg \cdot m^2}{s^2}}{1J}} = 512\,\frac{m}{s}$

Step 3: Check your work

Does your answer seem reasonable? Are sig. fig. correct?

The molar mass of N_2 is smaller than that of Ar, so the root-mean-square speed for N_2 should be greater than that of Ar (at the same temperature), but the molar mass difference is not enormous so the answers should be similar in magnitude.

2. The data provided can be used to calculate the molar mass of the gas. Hopefully, that will make it obvious what the gas can be. Remember that the calculated molar mass will be in kg/mol, so it will have to be converted into g/mol!

Step 1: Convert temperature into Kelvin

$T = 0 + 273.15 = 273.15\ K = 273\ K$

This value only has 3 sig. fig. because 0 ℃ has no decimal places; however, we will still use 273.15 K in the calculation to minimize rounding error.

Step 2: Rearrange the formula to isolate molar mass (M)

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

$$v_{rms}^2 = \frac{3RT}{M}$$

$$M = \frac{3RT}{v_{rms}^2}$$

Step 3: Solve for molar mass

$$M = \frac{3\left(8.314462\frac{J}{mol\cdot K}\right)(273.15K)}{\left(1838\frac{m}{s}\right)^2} \times \frac{1\frac{kg\cdot m^2}{s^2}}{1J} = 0.00202\,\frac{kg}{mol}$$

Step 4: Convert molar mass into g/mol

$$M = 0.00202\,\frac{kg}{mol} \times \frac{1000\ g}{1\ kg} = 2.02\,\frac{g}{mol}$$

Step 5: Identify the gas

There are only two gases that could possibly have a molar mass of 2.02 g/mol.

The sample must either be atoms of ^2H or it must be H_2.

Since atoms of ^2H would react to give molecules of $^2H_2$ (which has a molar mass of approximately 4 g/mol), that's not a reasonable option.

So, the answer must be H_2.

Step 6: Check your work

Does your answer seem reasonable? Did you get a molar mass that could reasonably correspond to an atom or small molecule? Getting a molar mass smaller than 1 g/mol would indicate that something went wrong in the calculation. Similarly, getting a molar mass larger than ~1,000 g/mol would indicate that something went wrong since molecules that large would tend to be solids (not gases) under most circumstances.