Answers to Practice Test Questions 3
Light and the Atom

1.

(a) Electrons in atoms exist in orbitals.

Energy of electrons in atoms is quantized.

(b) Energy of light is proportional to its frequency.

Light acts as a particle.

When this was used as a test question, many students reported observations relating to these experiments rather than conclusions. (e.g. "different elements emit different wavelengths of light" is an observation not a conclusion)

That said, there was more than one acceptable answer for each experiment. Two acceptable answers are listed above for each experiment.

2.

(a) When a surface is irradiated with electromagnetic radiation (light) of a high enough frequency, electrons are ejected from the atoms in the surface and a current flows.

(b) Light can act as a particle (i.e. a photon).

(c) If light behaved only as a wave, its energy would be increased by increasing the intensity of the light. If this were the case, the current measured would increase steadily as the intensity of light shone on the surface increased, and there would be no threshold frequency.

The fact that there is a threshold frequency indicates that the light energy is "packaged" into particles of light (which we call photons). When one photon of light hits the surface, it can excite an electron out of its atom only if the energy of the photon is high enough.

Increasing the intensity of the light increases the number of photons but does not increase the energy of any individual photon. Thus, above the threshold frequency, increasing the intensity of the light increases the number of electrons ejected from the surface but does not affect the kinetic energy of any of the electrons.

3. According to Rutherford's model of the atom, electrons in the diffuse cloud of electrons should be able to have any energy.

Each line in an atomic line spectrum is light which has the same energy (per photon) as the difference between the energies of two atomic orbitals.

If electrons could have any energy then it should be possible to have any energy difference between two atomic orbitals. This would not give individual "lines" of light (with specific energies); it would give the full spectrum. Since the full spectrum is not absorbed (or emitted), Rutherford's model must be wrong.

4. Successes of Bohr atomic theory

- The energy of each state (n = 1, 2, 3, etc.) of a hydrogen atom can be calculated.
- The average radius of a hydrogen atom in each state (n = 1, 2, 3, etc.) can be calculated.
- Experiments measuring these values show that the calculated values are correct.

Failures of Bohr atomic theory

- Angular momentum is not treated correctly.
- Electrons do not actually orbit at fixed distances from the nucleus.
- Calculations only work for single-electron species (hydrogen and the one-electron cations).

5. In emission spectroscopy, light is shone on a sample. This excites the sample into a higher energy state. When the sample relaxes back to a lower energy state, it releases energy in the form of light. This light is emitted in all directions and a fraction of it strikes a detector (carefully positioned so that it does not detect any of the light originally shone on the sample).

The detector measures which wavelengths of light are emitted. The energy corresponding to each of these wavelengths is equal to the energy gap between two different states in the sample.

Thus, emission spectroscopy allows us to determine the relative energies of the different energetic states possible for a sample.

Emission spectroscopy can also be used to identify an unknown sample as long as it matches a previously analyzed sample. The emission spectrum for each element, for example, is unique. So, if an unknown sample has the same emission spectrum as argon, we can conclude that it is also argon.

6.

(a) The threshold frequency is the minimum frequency of light for which a single photon will have enough energy to excite an electron out of an atom. Thus, the energy of this photon will be equal to the ionization energy of the metal. $\left(E_i = E_{photon}\right)$

$$E_{photon} = h\nu$$

$$E_{photon} = \left(6.626070 \times 10^{-34} \tfrac{J}{Hz}\right)(1.60 \times 10^{15} Hz) = 1.06 \times 10^{-18} J$$

Therefore the energy required to excite a single electron from a single atom in the metal is 1.06×10^{-18} J.

(b) $$E_i = 1.06 \times 10^{-18} \frac{J}{electron} \times \frac{1kJ}{1000J} \times \frac{6.022141 \times 10^{23} electrons}{1mol} = 638 \frac{kJ}{mol}$$

(c) all frequencies of light greater than 1.6×10^{15} Hz

7.

(a) This formula only applies to atoms (and ions) with only one electron.

(b) **Step 1: Convert the energy in kJ/mol to J/atom (or J/photon)**

$$E_{photon} = E_i = \left(418.81\frac{kJ}{mol}\right)\left(\frac{1000J}{1kJ}\right)\left(\frac{1mol}{6.022141\times10^{23}}\right) = 6.9545\times10^{-19}J$$

Step 2: Calculate frequency from energy

E = hν calculates energy of a <u>single</u> photon!

$$E_{photon} = h\nu$$

$$\nu = \frac{E_{photon}}{h} = \frac{6.9545\times10^{-19}J}{6.626070\times10^{-34}\frac{J}{Hz}} = 1.0496\times10^{15}Hz$$

Step 3: Calculate wavelength from frequency

$$c = \nu\lambda$$

$$\lambda = \frac{c}{\nu} = \frac{2.997925\times10^{8}\frac{m}{s}}{1.0496\times10^{15}Hz}\times\frac{1Hz}{1\frac{1}{s}} = 2.8563\times10^{-7}m$$

Step 4: Convert wavelength to nm

$$\lambda = 2.8563\times10^{-7}m\times\frac{10^{9}nm}{1m} = 285.63nm$$

Step 5: Check your work

Does your answer seem reasonable? Are sig. fig. correct?

The ionization energy has a similar order of magnitude to R_H and a wavelength in the UV range. (We'd expect UV, visible or IR.)

8.

(a) The excess energy from a photon (i.e. the energy that is not required to excite the electron out of the atom) is carried with the electron as kinetic energy. Therefore, we can calculate the energy required to excite an electron out of an atom by finding the difference between the energy of the photon and the kinetic energy of the ejected electron.

Step 1: Calculate the energy of one photon

$E = h\nu$ and $c = \lambda\nu$ therefore

$$E_{photon} = \frac{hc}{\lambda} = \frac{\left(6.626070\times10^{-34}\frac{J}{Hz}\right)\left(2.997925\times10^{8}\frac{m}{s}\right)}{(65nm)}\times\frac{10^{9}nm}{1m}\times\frac{1Hz}{1\frac{1}{s}} = 3.1\times10^{-18}J$$

Step 2: Calculate the energy required to excite one electron from an atom

$$E_{photon} = E_i + E_k$$

$$E_i = E_{photon} - E_k$$

$$E_i = (3.1\times10^{-18}J) - (2.0\times10^{-18}J) = 1.1\times10^{-18}J$$

This is the minimum amount of energy required for a photon to excite an electron from an atom in the metal. Therefore, this is the threshold energy for the metal.

Step 3: Check your work

Does your answer seem reasonable? Are sig. fig. correct?
The threshold energy has a similar order of magnitude to R_H.

(b) Since the metal is the same, the threshold energy is the same. To eject as many electrons as possible, we want to "waste" no energy as kinetic energy of ejected electrons. Each photon should have exactly enough energy to excite a single electron from a single atom. To do this, use light where the energy of one photon equals the metal's threshold energy.

Every photon can eject **one** electron from the metal. Calculate the number of photons in a beam of light with 7.33×10^{-15} J where each photon has an energy of 1.1×10^{-18} J to get the maximum number of electrons that could be ejected from the metal using the beam of light.

$$E_{beam} = \#photons \times E_{photon}$$

$$\#photons = \frac{E_{beam}}{E_{photon}} = \frac{7.33\times10^{-15}J}{1.1\times10^{-18}J} = 6.9 \times 10^3$$

Since the beam can contain, at most, 6.9×10^3 photons (and still be able to excite electrons out of the metal), it can eject at most 6.9×10^3 electrons from the metal.

9. The lowest energy light will correspond to the smallest energy gap. The smallest energy gap between $n = 5$ and a higher energy level is the gap between $n = 5$ and $n = 6$.

A hydrogen atom has one proton so $Z = 1$.

Step 1: Calculate the energy for H with *n* = 5

$$E_{n=5} = -R_H \frac{Z^2}{n^2} = -R_H \frac{(1)^2}{(5)^2} = -\frac{1}{25} R_H = -8.719488\times10^{-20} J$$

Step 2: Calculate the energy for H with *n* = 6

$$E_{n=6} = -R_H \frac{Z^2}{n^2} = -R_H \frac{(1)^2}{(6)^2} = -\frac{1}{36} R_H = -6.055200\times10^{-20} J$$

Step 3: Calculate the difference in energy between the *n* = 5 and *n* = 6 states ($\Delta E = E_{final} - E_{initial}$)

$$\Delta E = E_{n=5} - E_{n=6} = (-8.719488 \times 10^{-20}\, J) - (-6.055200 \times 10^{-20}\, J)$$

$$\Delta E = -2.664288 \times 10^{-20}\, J$$

Step 4: Calculate the energy of the photon released

Since the sign of ΔE is negative, energy is released.

The energy of the photon released is 2.664288×10^{-20}J.

Step 5: Calculate the frequency of the photon released

$$\boldsymbol{E = h\nu}$$

$$\boldsymbol{\nu = \frac{E}{h} = \frac{2.664288\times10^{-20}J}{6.626070\times10^{-34}\frac{J}{Hz}} \times 4.020917 \times 10^{13} Hz}$$

Step 6: Calculate the wavelength of the photon released

$$\boldsymbol{c = \nu\lambda}$$

$$\boldsymbol{\lambda = \frac{c}{\nu} = \frac{2.997925\times10^{8}\frac{m}{s}}{4.020917\times10^{13}Hz} \times \frac{1Hz}{1\frac{1}{s}} = 7.455823 \times 10^{-6} m}$$

$$\boldsymbol{\lambda = 7.455823 \times 10^{-6} m \times \frac{10^6 \mu m}{1m} = 7.455823 \mu m}$$

Steps 5 and 6 may be combined using E=hc/λ. <u>Always</u> check your work!

10.

(a) 628 nm

Energy of a photon is directly proportional to frequency and inversely proportional to wavelength. As such, the shorter wavelength of light corresponds to higher energy photons.

(b) The ionization energy for a ground state hydrogen atom is equal to the energy difference between an electron with $n = 1$ and an electron with $n = \infty$ (the ionization limit). The energy for an electron with $n = \infty$ is 0 J.

A hydrogen atom has one proton so $Z = 1$.

Step 1: Calculate the energy for a hydrogen atom with *n* = 1

$$E_{n=1} = -R_H \frac{Z^2}{n^2} = -R_H \frac{(1)^2}{(1)^2} = -R_H = -2.179872 \times 10^{-18} J$$

Step 2: Calculate the ionization energy for a ground state hydrogen atom

$$E_i = E_{n=\infty} - E_{n=1} = (0J) - (-2.179872 \times 10^{-18} J) = 2.179872 \times 10^{-18} J$$

Step 3: Check your work

Does your answer seem reasonable? Are sig. fig. correct?

The ionization energy of ground state hydrogen is R_H.

(c) To answer this question, it is necessary to calculate the energy of the laser light. Since 628 nm is the higher energy wavelength, calculate its energy first.

$$E = h\nu = \frac{hc}{\lambda} = \frac{\left(6.626070 \times 10^{-34} \frac{J}{Hz}\right)\left(2.997925 \times 10^{8} \frac{m}{s}\right)}{(628nm)} \times \frac{10^9 nm}{1m} = 3.16 \times 10^{-19} J$$

The energy required to excite an electron out of a ground state hydrogen atom is almost ten times as large as the energy provided by a photon of 628 nm light. Therefore, neither wavelength of light produced by the ruby laser would be energetic enough to ionize the hydrogen atom.

Alternately, part (c) could be solved by calculating the wavelength of light corresponding to the energy required to ionize H ($E = hc/\lambda$ gives λ = 91.16 nm). This is the longest wavelength of light which will ionize a hydrogen atom. Therefore, the ruby laser will not be able to do so as both wavelengths it produces are longer (lower energy) than 91.16 nm.

11.

(a) Li^{2+} starts in the ground state (n = 1).

The lowest energy photon that Li^{2+} in the n = 1 state can absorb will excite the ion into the n = 2 state.

Li^{2+} has three protons, so Z = 3.

Step 1: Calculate the energy for Li^{2+} with *n* = 1

$$E_{n=1} = -R_H \frac{Z^2}{n^2} = -R_H \frac{(3)^2}{(1)^2} = -9R_H = -1.961885 \times 10^{-17}\,J$$

Step 2: Calculate the energy for Li^{2+} with *n* = 2

$$E_{n=2} = -R_H \frac{Z^2}{n^2} = -R_H \frac{(3)^2}{(2)^2} = -\frac{9}{4}R_H = -4.904712 \times 10^{-18}\,J$$

Step 3: Calculate the difference in energy between the *n* = 1 and *n* = 2 states ($\Delta E = E_{final} - E_{initial}$)

$$\Delta E = E_{n=2} - E_{n=1} = (-4.904712 \times 10^{-18}\,J) - (-1.961885 \times 10^{-17}\,J)$$

$$\Delta E = 1.471414 \times 10^{-17}\,J$$

Step 4: Calculate the energy of the photon absorbed

Since the sign of ΔE is positive, energy is absorbed.

The energy of the photon absorbed is 1.471414×10^{-17}J.

Step 5: Check your work

Does your answer seem reasonable? Are sig. fig. correct?

The energy of the photon has a similar order of magnitude to R_H.

(b) The energy of the photon would be smaller.

As n increases, the energies of the states get closer together, so the energy gap between n = 3 and n = 4 is smaller than the energy gap between n = 1 and n = 2.

12. A hydrogen atom has one proton so Z = 1.

Step 1: Calculate the energy of one photon

$E = h\nu$ and $c = \lambda\nu$ therefore

$$E = \frac{hc}{\lambda} = \frac{\left(6.626\times10^{-34}\,\tfrac{J}{Hz}\right)\left(2.9979\times10^{8}\,\tfrac{m}{s}\right)}{(1940nm)} \times \frac{10^{9}\,nm}{1m} \times \frac{1Hz}{1\tfrac{1}{s}} = 1.024\times10^{-19}\,J$$

Step 2: Calculate the energy of hydrogen in the *n* = 8 state

$E_n = -R_H \frac{Z^2}{n^2}$ therefore

$$E_{n=8} = -\left(2.179\times10^{-18}\,J\right)\frac{(1)^2}{(8)^2} = -3.405\times10^{-20}\,J$$

Step 3: Calculate the energy of hydrogen in the *n* = ??? state (after emission)

$E_{photon} = E_{n=8} - E_{n=???}$ therefore

$E_{n=???} = E_{n=8} - E_{photon} = \left(-3.405 \times 10^{-20} J\right) - \left(1.024 \times 10^{-19} J\right) = -1.364 \times 10^{-19} J$

Step 4: Calculate *n* for the final state of the hydrogen atom

$$E_n = -R_H \frac{Z^2}{n^2} \text{ therefore } n^2 = -R_H \frac{Z^2}{E_n} \text{ therefore}$$

$$n = \sqrt{-R_H \frac{Z^2}{E_n}} = \sqrt{-\left(2.179 \times 10^{-18} J\right) \frac{(1)^2}{\left(-1.364 \times 10^{-19} J\right)}} = 4$$

Step 5: Check your work

Does your answer seem reasonable? Are sig. fig. correct?

The answer was a small integer (or rounds easily to one).

13.

(a) The ionization energy for ground state He^+ is equal to the energy difference between an electron with $n = 1$ and an electron with $n = \infty$ (the ionization limit). The energy for an electron with $n = \infty$ is 0 J.

He^+ has two protons so $Z = 2$.

Step 1: Calculate the energy for He^+ with *n* = 1

$$E_{n=1} = -R_H \frac{Z^2}{n^2} = -R_H \frac{(2)^2}{(1)^2} = -4R_H = -8.719488 \times 10^{-18} J$$

Step 2: Calculate the ionization energy for ground state He^+

$$E_i = E_{n=\infty} - E_{n=1} = (0J) - (-8.719488 \times 10^{-18} J) = 8.719488 \times 10^{-18} J$$

Step 3: Check your work

Does your answer seem reasonable? Are sig. fig. correct?

The ionization energy has a similar order of magnitude to R_H.

(b) **Step 1: Calculate the energy of the photon**

$$E_{photon} = h\nu$$

$$E_{photon} = \left(6.626070 \times 10^{-34} \frac{J}{Hz}\right)\left(4.776 \times 10^{16} Hz\right)$$

$$E_{photon} = 3.165 \times 10^{-17} J$$

Step 2: Calculate the energy "left over" after photon excites electron from He^+

Any extra energy from the photon will be transferred as kinetic energy to the electron. Therefore, the kinetic energy of the electron is:

$$E_{kinetic} = E_{photon} - E_i$$

$$E_{kinetic} = 3.165\times10^{-17}\,J - 8.719488\times10^{-18}\,J$$

$$E_{kinetic} = 2.293\times10^{-17}\,J$$

Step 3: Check your work

Does your answer seem reasonable? Are sig. fig. correct?

(c) The ionization of a hydrogen atom would be less than that of a helium cation. Both have 1 electron but the helium cation has two protons while the hydrogen atom only has 1. The electron in the hydrogen atom experiences a smaller effective nuclear charge and therefore is easier to remove.