# Answers to Exercise 12.4
# More Weak Acid Calculations

1.

(a) **Step 1: Calculate nominal concentration of solution (initial concentration of acid)**

$1.25\ g\ \times \frac{1\ mol}{43.028\ g} = 0.0291\ mol$

$\frac{0.0291\ mol}{1.000\ L} = 0.0291\ M$

**Step 2: Calculate $K_a$ from $pK_a$**

$K_a = 10^{-pK_a} = 10^{-4.72} = 1.9 \times 10^{-5}$

**Step 3: Write a balanced chemical equation and organize all known information**

| | $HA_{(aq)}$ | $\rightleftharpoons$ | $H^+_{(aq)}$ | $+$ | $A^-_{(aq)}$ |
|---|---|---|---|---|---|
| I (M) | $0.0291$ | | $10^{-7}$ | | $0$ |
| C (M) | $-x$ | | $+x$ | | $+x$ |
| E (M) | $0.0291 - x \approx 0.0291$ | | $10^{-7} + x \approx x$ | | $x$ |

**Step 4: Write equilibrium constant expression and calculate x**

$K_a = \frac{(a_{H^+})(a_{A^-})}{a_{HA}}$

$1.9 \times 10^{-5} = \frac{(x)(x)}{0.0291}$

$(1.9 \times 10^{-5})(0.0291) = x^2$

$x = \sqrt{(1.9 \times 10^{-5})(0.0291)}$

$x = 7.4 \times 10^{-4}$

**Step 5: Check assumptions. If invalid, repeat calculation without the invalid assumption.**

Assumption #1: $0.0291 - x \approx 0.0291$ $\quad 0.0291 - 0.00074 = 0.0283$ close but not quite

Assumption #2: $10^{-7} + x \approx x$ $\quad 10^{-7} + 0.00074 = 0.00074$ assumption okay

**Step 6: Try again without the first assumption (keep second one)**

$K_a = \frac{(a_{H^+})(a_{A^-})}{a_{HA}}$

$1.9 \times 10^{-5} = \frac{(x)(x)}{(0.0291-x)}$

$(1.9 \times 10^{-5})(0.0291 - x) = x^2$

$5.5 \times 10^{-7} - (1.9 \times 10^{-5})x = x^2$

$x^2 + (1.9 \times 10^{-5})x - (5.5 \times 10^{-7}) = 0$

*Use quadratic equation (or other tools) to solve, giving:*

$x = 7.3 \times 10^{-4}$

*($x = -7.5 \times 10^{-4}$ would give negative concentrations for $H^+$ and $A^-$ which is impossible)*

**Step 7: Calculate pH from $a_{H^+}$**

$a_{H^+} = x = 7.3 \times 10^{-4}$

$pH = -log(a_{H^+}) = -log(7.3 \times 10^{-4}) = 3.13$

**Step 8: Check your work**

*It's an acid, so expect a pH below 7. If this had been a strong acid, the pH would have been $-log(0.0291) = 1.537$. Since it's a weak acid, it makes sense that the pH is higher.*

*Note that if we had stuck with both initial assumptions, we'd have calculated $pH = -log(7.4 \times 10^{-4}) = 3.13$. So, maybe we could have left it as "close enough" after all…*

(b) **Step 1: Calculate nominal concentration of solution (initial concentration of acid)**

$1.25\ g \times \frac{1\ mol}{68.4594\ g} = 0.0183\ mol$

$\frac{0.0183\ mol}{1.000\ L} = 0.0183\ M$

**Step 2: Calculate $K_a$ from $pK_a$**

$K_a = 10^{-pK_a} = 10^{-1.96} = 1.1 \times 10^{-2}$

**Step 3: Write a balanced chemical equation and organize all known information**

| | $HA_{(aq)}$ | $\rightleftharpoons$ | $H^+_{(aq)}$ | $+$ | $A^-_{(aq)}$ |
|---|---|---|---|---|---|
| I (M) | $0.0183$ | | $10^{-7}$ | | $0$ |
| C (M) | $-x$ | | $+x$ | | $+x$ |
| E (M) | $0.0183 - x$ | | $10^{-7} + x \approx x$ | | $x$ |

**Step 4: Write equilibrium constant expression and calculate x**

$K_a = \frac{(a_{H^+})(a_{A^-})}{a_{HA}}$

$1.1 \times 10^{-2} = \frac{(x)(x)}{(0.0183 - x)}$

$(1.1 \times 10^{-2})(0.0183 - x) = x^2$

$2.0 \times 10^{-4} - (1.1 \times 10^{-2})x = x^2$

$x^2 + (1.1 \times 10^{-2})x - 2.0 \times 10^{-4} = 0$

*Use quadratic equation (or other tools) to solve, giving:*

$x = 9.7 \times 10^{-3}$

*($x = -2.1 \times 10^{-2}$ would give negative concentrations for $H^+$ and $A^-$ which is impossible)*

**Step 5: Check assumption. If invalid, repeat calculation without it.**

Assumption: $10^{-7} + x \approx x$ $\quad$ $10^{-7} + 0.0097 = 0.0097$ $\quad$ assumption okay

**Step 6: Calculate pH from $a_{H^+}$**

$a_{H^+} = x = 9.7 \times 10^{-3}$

$pH = -log(a_{H^+}) = -log(9.7 \times 10^{-3}) = 2.01$

**Step 8: Check your work**

*It's an acid, so expect a pH below 7. If this had been a strong acid, the pH would have been $-log(0.0183) = 1.739$. Since it's a weak acid, it makes sense that the pH is higher.*

(c) **Step 1: Calculate nominal concentration of solution (initial concentration of acid)**

$1.25\ g\ \times \frac{1\ mol}{63.0128\ g} = 0.0198\ mol$

$\frac{0.0198\ mol}{1.000\ L} = 0.0198\ M$

**Step 2: Calculate $\boldsymbol{K_a}$ from $\boldsymbol{pK_a}$**

$K_a = 10^{-pK_a} = 10^{-3.39} = 4.1 \times 10^{-4}$

**Step 3: Write a balanced chemical equation and organize all known information**

| | $HA_{(aq)}$ | $\rightleftharpoons$ | $H^+_{(aq)}$ | + | $A^-_{(aq)}$ |
|---|---|---|---|---|---|
| I (M) | $0.0198$ | | $10^{-7}$ | | $0$ |
| C (M) | $-x$ | | $+x$ | | $+x$ |
| E (M) | $0.0198 - x \approx 0.0198$ | | $10^{-7} + x \approx x$ | | $x$ |

**Step 4: Write equilibrium constant expression and calculate x**

$K_a = \frac{(a_{H^+})(a_{A^-})}{a_{HA}}$

$4.1 \times 10^{-4} = \frac{(x)(x)}{0.0198}$

$(4.1 \times 10^{-4})(0.0198) = x^2$

$x = \sqrt{(4.1 \times 10^{-4})(0.0198)}$

$x = 2.8 \times 10^{-3}$

**Step 5: Check assumptions. If invalid, repeat calculation without the invalid assumption.**

| | | |
|---|---|---|
| Assumption #1: $0.0198 - x \approx 0.0198$ | $0.0198 - 0.0028 = 0.0170$ | not close enough |
| Assumption #2: $10^{-7} + x \approx x$ | $10^{-7} + 0.0028 = 0.0028$ | assumption okay |

**Step 6: Try again without the first assumption (keep second one)**

$K_a = \frac{(a_{H^+})(a_{A^-})}{a_{HA}}$

$4.1 \times 10^{-4} = \frac{(x)(x)}{(0.0198 - x)}$

$(4.1 \times 10^{-4})(0.0198 - x) = x^2$

$5.5 \times 10^{-7} - (8.1 \times 10^{-6})x = x^2$

$x^2 + (4.1 \times 10^{-4})x - (8.1 \times 10^{-6}) = 0$

*Use quadratic equation (or other tools) to solve, giving:*

$x = 2.6 \times 10^{-3}$

*($x = -3.1 \times 10^{-3}$ would give negative concentrations for $H^+$ and $A^-$ which is impossible)*

**Step 7: Calculate pH from $\boldsymbol{a_{H^+}}$**

$a_{H^+} = x = 2.6 \times 10^{-3}$

$pH = -log(a_{H^+}) = -log(2.6 \times 10^{-3}) = 2.58$

**Step 8: Check your work**

*It's an acid, so expect a pH below 7. If this had been a strong acid, the pH would have been $-log(0.0198) = 1.703$. Since it's a weak acid, it makes sense that the pH is higher.*

2. General approach: Use the equilibrium constant expression, but this time, the activity of $H^+$ can be calculated directly from the pH provided.

**Step 1: Calculate $K_a$ from $pK_a$**

$K_a = 10^{-pK_a} = 10^{-3.39} = 4.1 \times 10^{-4}$

**Step 2: Calculate $a_{H^+}$ from $pH$**

$a_{H^+} = 10^{-pH} = 10^{-2.00} = 1.0 \times 10^{-2}$

**Step 3: Write a balanced chemical equation and organize all known information**

| | $HA_{(aq)}$ | $\rightleftharpoons$ | $H^+_{(aq)}$ | $+$ | $A^-_{(aq)}$ |
|---|---|---|---|---|---|
| I (M) | $???$ | | $10^{-7}$ | | $0$ |
| C (M) | $-x$ | | $+x$ | | $+x$ |
| E (M) | $???-x \approx ???$ | | $10^{-7}+x \approx x$ | | $x$ |

Since we don't know $a_{HA}$, start by assuming that both assumptions will be valid.

$a_{H^+} = 1.0 \times 10^{-2}$ therefore $x = 1.0 \times 10^{-2}$

**Step 4: Write equilibrium constant expression and calculate $a_{HA}$**

$K_a = \frac{(a_{H^+})(a_{A^-})}{a_{HA}}$

$4.1 \times 10^{-4} = \frac{(1.0\times10^{-2})(1.0\times10^{-2})}{a_{HA}}$

$a_{HA} = \frac{(1.0\times10^{-2})(1.0\times10^{-2})}{4.1\times10^{-4}} = 0.25$

**Step 5: Check assumptions. If invalid, repeat calculation without the invalid assumption.**

Assumption #1: $0.25 - x \approx 0.25$ $\quad 0.25 - 0.01 = 0.24$ $\quad$ close enough*

Assumption #2: $10^{-7} + x \approx x$ $\quad 10^{-7} + 0.01 = 0.01$ $\quad$ assumption okay

*If you were to repeat the calculation without making the assumption, you'd get the same answer once rounded to the correct number of sig. fig. The solution to 1(a) demonstrated how this is possible.

**Step 6: Calculate concentration of acid from activity**

$M_{HA} = a_{HA} \times 1M = 0.25\ M$

**Step 7: Check your work**

*Does your answer seem reasonable?*

*The solution has a higher concentration than in question 1(c) and therefore has a lower pH.*

3. General approach: Use the equilibrium constant expression, but this time, the activity of $H^+$ can be calculated directly from the pH provided. Once you know the concentration of acid required, use stoichiometry to calculate the mass of acid required to make the solution.

**Step 1: Calculate $K_a$ from $pK_a$**

$K_a = 10^{-pK_a} = 10^{-4.72} = 1.9 \times 10^{-5}$

**Step 2: Calculate $a_{H^+}$ from $pH$**

$a_{H^+} = 10^{-pH} = 10^{-2.72} = 1.9 \times 10^{-3}$

**Step 3: Write a balanced chemical equation and organize all known information**

| | $HA_{(aq)}$ | $\rightleftharpoons$ | $H^+_{(aq)}$ | $+$ | $A^-_{(aq)}$ |
|---|---|---|---|---|---|
| I (M) | ??? | | $10^{-7}$ | | 0 |
| C (M) | $-x$ | | $+x$ | | $+x$ |
| E (M) | $??? - x \approx ???$ | | $10^{-7} + x \approx x$ | | $x$ |

Since we don't know $a_{HA}$, start by assuming that both assumptions will be valid.

$a_{H^+} = 1.9 \times 10^{-3}$ therefore $x = 1.9 \times 10^{-3}$

**Step 4: Write equilibrium constant expression and calculate $a_{HA}$**

$$K_a = \frac{(a_{H^+})(a_{A^-})}{a_{HA}}$$

$$1.9 \times 10^{-5} = \frac{(1.9 \times 10^{-3})(1.9 \times 10^{-3})}{a_{HA}}$$

$$a_{HA} = \frac{(1.9 \times 10^{-3})(1.9 \times 10^{-3})}{1.9 \times 10^{-5}} = 0.19$$

**Step 5: Check assumptions. If invalid, repeat calculation without the invalid assumption.**

| | | |
|---|---|---|
| Assumption #1: $0.19 - x \approx 0.19$ | $0.19 - 0.0019 = 0.19$ | assumption okay |
| Assumption #2: $10^{-7} + x \approx x$ | $10^{-7} + 0.0019 = 0.0019$ | assumption okay |

**Step 6: Calculate concentration of acid from activity**

$M_{HA} = a_{HA} \times 1M = 0.19\ M$

**Step 7: Calculate moles of acid required to make solution**

$$n_{HA} = 250.\, mL \times 0.19\ \frac{mol}{L} \times \frac{1\ L}{1000\ mL} = 0.048\ mol$$

**Step 8: Calculate mass of acid required to make solution**

$$m_{HA} = 0.048\ mol \times \frac{43.028\ g}{1\ mol} = 2.0\ g$$

**Step 9: Check your work**

*Does your answer seem reasonable?*

*The solution has a higher concentration than in question 1(a) and therefore has a lower pH.*